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CASH-IN-ADVANCE CONSTRAINT ON R&D IN A SCHUMPETERIAN GROWTH MODEL WITH AN ENDOGENOUS MARKET STRUCTURE

Chienyu Huang
(Southwestern University of Finance and Economics)

Juin-Jen Chang
(Institute of Economics, Academia Sinica)

Lei Ji
(OFCE Sciences-Po and SKEMA Business School)

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Chienyu Huang, Southwestern University of Finance and Economics

Juin-Jen Chang,*Institute of Economics, Academia Sinica

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Abstract

In this paper we explore the effects of monetary policy on the number of firms, firm market size, inflation and growth in a Schumpeterian growth model with endogenous market structure and cash-in-advance (CIA) constraints on two distinct types of R&D investment – in-house R&D and entry investment. This allows us to match the empirical evidence and provides novel implications to the literature. We show that if in-house R&D (quality improvement-type R&D) is subject to the CIA constraint, raising the nominal interest rate increases the number of firms and inflation, but decreases the firm size and economic growth. By contrast, if entry investment (variety expansion-type R&D) is subject to the CIA constraint, these variables adversely respond to such a monetary policy. Besides, our model generates rich transitional dynamics in response to a change in monetary policy, when R&D/entry is restricted by a cash constraint.

JEL classication: O30, O40, E41.

Keywords: CIA constraints on R&D, endogenous market structure, monetary policy, economic growth.

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*Author for correspondence: Juin-Jen Chang, Institute of Economics, Academia Sinica, Taipei115, Taiwan; Tel: +886 2 27822791 ext. 532; Fax: +886 2 27853946; Email: jjchang@econ.sinica.edu.tw.

1 Introduction

This paper explores the long-run and short-run effects of monetary policy in a Schumpeterian growth model through CIA constraints on two types of R&D investment, namely, in-house R&D and entry investment (and on consumption in some cases). Our analytical framework is a "second generation" endogenous growth model with an "endogenous market structure" developed by Peretto (1998a). This is the first time a second-generation growth model has been used to study the macro consequences of monetary policy by shedding light on the importance of distinct cash constraints on R&D investments.[1] The endogeneity of the market structure provides novel implications and new insights to the relevant literature.

The model is built to be consistent with two major sets of facts. First, the empirical evidence (e.g., Hall 1992 and Himmelberg and Petersen 1994) reports a strong R&D-cash flow sensitivity for firms. R&D-intensive firms are required to hold cash in order to smooth their R&D spending over time.[2] In particular, cash holdings have a stronger impact on R&D in younger firms, which are more likely to face binding financing constraints (see Brown and Petersen 2009, 2011 and Brown et al. 2012). Importantly, relative to the traditional physical investment, R&D activities exhibit a stronger investment-cash flow sensitivity. Given that the conventional monetary model only focuses on the CIA constraint on consumption and physical investment (e.g., Stockman 1981 and Wang and Yip 1992), the lack of an appropriate consideration of a CIA constraint on R&D investments may fail to not only reflect reality, but also to provide a complete picture for the implications of monetary policy. Second, our model is consistent with the fact of industrial organization (IO) in the sense that the in-house R&D activities depend on firm market size which is endogenously determined through the market structure (see Cohen and Klepper 1996 a,b and Adams and Jaffe 1996). Models that come out of the first wave of endogenous growth theory (in which each individual firm's market size is exogenous and equal to the entire economy) do not correspond to the IO realities and, as a result, give rise to inappropriate predictions, such as the "scale effect," that are inconsistent with the data (Backus et al. 1992).[3] Therefore, one should be cautious as the results of the long-run monetary effects are based on the Schumpeterian growth model without an endogenous market structure, such

[1]While Chu and Ji (2012) also use a second-generation growth model to examine the growth effect of monetary policy, their analysis is confined in a Lucasian (1980) CIA economy without any cash constraint on R&D. While Chu and Cozzi (2013) analyze a CIA constraint on R&D, their analysis is based on a quality-ladder R&D model with a fixed market structure, rather than a second-generation model with an endogenously-determined market structure.

[2]For recent observations, the reader can refer to Harhoff (1998), Hall et al. (1999), Mulkay et al. (2001), Brown et al. (2009, 2012), and Brown and Petersen (2009, 2011), among others.

[3]The so-called "scale effect" is a positive relation between the economy's size and its *growth rate*, which is rejected by actual data. In a second-generation growth model, new entrants are allowed to compete against incumbents and reduce the individual firm's market share. Since individual firms' markets may change in response to entry and exit and are not equivalent to that of the whole economy, the scale effect can be eliminated.

as in Funk and Kromen (2010) and Chu and Lai (2013).

In the model, market structure is measured by the number of firms and firm market size, as in Peretto (1998a). By endogenizing the size and number of firms, the market structure shapes the profit-seeking firm's behavior by affecting the returns to innovation and entry. To characterize both market structure and innovation simultaneously, our model has both horizontal (variety expansion) and vertical (quality improvement) dimensions of technology space.[4] In the vertical dimension, incumbents reduce production cost by conducting in-house R&D. In the horizontal dimension, new entrants compete with incumbents by bringing a new product into the market. Since the quality-improved R&D interacts with the variety-expanded entry and this interaction is strongly related to the CIA constraint, market structure and economic growth are responsive to the government's monetary policy. To be more specific, various CIA constraints have rather different effects on the incumbent's R&D and the entrepreneur's entry, which in turn affect the firm size and the market concentration. This endogenously-determined market structure induces feedbacks that lead to different implications of monetary policy for economic growth from the conventional notion.

In this paper, we study both the long-run steady-state and the short-run transition effects of an increase in the nominal interest rate. In terms of the steady-state effects, we show that if in-house R&D (resp. entry investment) is subject to the CIA constraint, raising the nominal interest rate increases (resp. decreases) the number of firms, but decreases (resp. increases) the firm size, i.e., the number of workers per firm, and the total factor productivity (TFP) growth.

Why could an identical monetary policy end up with such different macro consequences? The economic intuition is straightforward. A higher nominal interest rate raises the cost of holding money, thus reducing the real money balances in the economy. If the money balances are required to engage in in-house R&D and entry investment is not restricted by such a constraint, in-house R&D becomes more expensive, compared to firm entry. Thus, the rents available to incumbents (quality improvement-type innovators) decrease, while the rents available to entrants (variety expansion-type innovators) increase. As a result, the number of firms expands, while the firm size shrinks, thus resulting in a lower rate of innovation and economic growth. However, if entry investment, instead of in-house R&D, is subject to the CIA constraint, raising the nominal interest rate restricts the variety-expanded innovation. Since the resource shifts away from entry to in-house R&D, the firm size increases and the market size decreases. An expansion in the firm size motivates the firms to engage in more R&D investment, and therefore the TFP growth rate rises in response.[5] The Schumpeterian paradigm indicates that, in favor of

[4]As stressed by Peretto and Connolly (2007) and Ji (2012), the traditional growth model with only one dimension of innovation is not able to model both innovation and market structure.

[5]A large body of empirical research has indicated that larger size can foster productivity growth because it allows firms to take advantage of the increasing returns associated with R&D. See, for example, Cohen and

economic growth, some extent of monopoly power is needed to act as the reward accruing to the successful firms from their innovative activities. In this monetary version of the Schumpeterian model, a rise in the nominal interest rate renders the existing innovators with such an extent of monopoly power by increasing entry costs and hence enhancing economic growth, if entry investment is subject to a larger cash constraint. This seems to be empirically plausible; R&D is in practice more likely to be liquidity constrained for younger firms or entrants.

The steady-state inflation effect conforms to the Fisher equation prescribe, regardless of whether in-house R&D or entry is constrained by money balances. In the long run, increasing the nominal interest rate requires an increase in the rate of money growth and, consequently, a higher nominal interest rate is associated with a higher inflation rate. This inflation effect, together with the growth effect above, give rise to novel economic implications. First, our model identifies a new channel that characterizes a positive effect of inflation on economic growth (i.e., the Mundell-Tobin effect). A particular emphasis is that our result is purely via the liquidity constraint and market-structure adjustment, rather than the conventional asset-substitution effect, stressed by Mundell (1963) and Tobin (1965). Second, due to the variety of the CIA constraints, our analysis predicts a *mixed* long-run relationship between growth and inflation, which reconciles the recent empirical findings. While studies by Fisher (1983) and Cooley and Hansen (1989) report a negative relationship between steady inflation and output/growth across countries, recent works by Bullard and Keating (1995), Bruno and Easterly (1998), and Ahmed and Rogers (2000) seemingly find no robust or even positive correlation in low-inflation industrialized economies. The recent evidence refers to a *non-monotonic* relationship, suggesting that the real output/growth effect of inflation is largely insignificant except for high-inflation countries.[6] Moreover, in our model money superneutrality is not valid, which is also consistent with empirical findings, such as in Fisher and Seater (1993).

Besides, monetary policy also leads to rich transitional dynamics when the market structure is endogenized. Our transition analysis shows that in response to a higher targeting level of the nominal interest rate, the firm size and the TFP growth both monotonically decrease (resp. increase) to the steady-state value, if in-house R&D (resp. entry) is subject to the CIA constraint. However, in either case, along the transition path the consumption growth rate, the employment rate, and the consumption expenditure may mis-adjust from their long-run steady states. Interestingly, when the cash constraints on in-house R&D, entry investment, and consumption all play a role in this story, the TFP growth rate may also exhibit a mis-adjustment in transition. In response to a higher nominal interest rate, the TFP growth rate

Klepper (1996a) and more recently Pagano and Schivardi (2003).

[6]More recently, Vaona (2012) finds that the intertemporal elasticity of substitution of working time is a key parameter for the shape of the inflation-growth nexus. Inflation has a negative effect on growth, provided that it is greater than zero.

rises in the long run, while it falls during the transition.

In a closely-related paper, Chu and Cozzi (2013) elegantly use two versions of the quality-ladder model to point out a mixed growth effect, as in our study.[7] However, our paper differs from theirs in three significant respects. First, while the growth effect is similar, the mechanism behind the result is quite different. In their model, R&D has only a vertical dimension, while our paper highlights the endogeneity of the market structure, which consists of both vertical and horizontal R&D. This salient model property enables us not only to differentiate the monetary implication for the quality-improved and variety-expanded R&D, but also to uncover the effects on the market and firm sizes. Second, our study predicts that an identical monetary policy could end up with very different market structures. Given that the industrial distribution of firm size has been shown to be crucial to both the short-run business cycle (e.g., Bernanke et al. 1996) and the long-run economic growth (e.g., Pagano and Schivardi 2003), this prediction then provides an empirically testable hypothesis concerning the relationship between market concentration and monetary policy. Third, they focus on the welfare implications with particular emphasis on the issues concerning a zero-interest-rate policy and over(under)investment in R&D, while we focus on the growth implications with an additional analysis of the transitional effect.

The rest of the paper proceeds as follows. Section 2 lays out the basic model. Section 3 constructs the general equilibrium, and discusses how monetary policy affects the economy in both the short run and long run through CIA constraints. Section 4 then concludes.

2 The Model

There is a monetary variant of the Peretto (1998a) model with CIA constraints on in-house R&D, entry investment, and consumption. The economy consists of households, firms (incumbents and entrants), and a government (solely represented by the monetary authority). Time t is continuous. For compact notation, the time index is suppressed throughout the paper.

2.1 Households

Consider an economy with a population growth rate λ, which is associated with a population size L. Each household chooses consumption C and leisure $(1-l)$ (where 1 is the normalized time and l are working hours) to maximize the following discounted sum of future instantaneous utilities:

[7] One is a scale-invariant model with an exogenous market structure and the other is a semi-endogenous growth model.

$$U(t) = \int e^{-(\rho-\lambda)}[lnC + \gamma ln(1-l)]dt, \tag{1}$$

subject to the budget constraint

$$\dot{A} + \dot{M}_m = (r-\lambda)A + l + iB + T - (\pi+\lambda)M_m - E, \tag{2}$$

and the CIA constraint

$$\xi_c E + B \leq M_m, \tag{3}$$

where $M_m = \frac{M_L}{P_m L}$. In line with Peretto (1998a), the price of labor is a numeraire. By defining M_L as the nominal money balances, P_m can then be viewed as the price of money in terms of labor and, accordingly, M_m is the real money balances per capita.[8] Thus, all quantity (non-price) variables are real and in per capita terms: A is real asset holdings, M_m is real money holdings, B is the real loans for R&D activities, E is the real consumption expenditure per capita, and T is the real lump-sum transfer from the government. Moreover, r is the real interest rate, i is the nominal interest rate, π is the inflation rate, and ρ is a constant time preference rate.

The CIA constraint (3) indicates that the real money balances M_m held by the households is required not only to purchase consumption goods E, but also to finance the firms' investment B. The parameter ξ_c is the weight of consumption on the cash constraint; in the case where $\xi_c \equiv 0$, only firms' investment is subject to the CIA constraint. The term B can be simply thought of as one-period loans, which are used to finance either the incumbent firms' in-house (quality improvement-type) R&D or new firms' entry investment (variety expansion-type R&D). As will be clear in Subsection 2.2, the amount of B crucially depends on how much the R&D and entry is restricted by the cash constraint. The specification of one-period loans is similar to Williamson (1987) and, accordingly, iB is then the interest rate payment on the loan for R&D activities. In Subsection 2.4, when deriving the no-arbitrage condition between this loan and other assets (i.e., $i = r + \pi$), we can see that the loan rate i is also the nominal interest rate.

Households consume all differentiated intermediate goods, and hence we set the bundle of consumption C as a CES combination of N types of intermediate goods. Let c_i be the consumption of the intermediate good j and ϵ be the elasticity of substitution. Thus, we can specify:

[8] The choice of deflator does not alter our main results.

$$C = [\int_{o}^{N} c_j^{(\epsilon-1)/\epsilon} dj]^{\frac{\epsilon}{\epsilon-1}}. \tag{4}$$

By denoting p_j as the price for intermediate good j in terms of labor, then the expenditure per capita is:

$$E = \int_{o}^{N} p_j c_j dj. \tag{5}$$

Define η and ψ as the multipliers associated with (2) and (3). Thus, the first-order conditions necessary for the household's optimization problem are given by:

$$\frac{1}{C} = P_C(\eta + \psi \xi_c),$$

$$\frac{\gamma}{1-l} = \eta,$$

$$\psi = \eta i,$$

$$-\eta(\pi + \lambda) + \psi = -\dot{\eta} + \eta(\rho - \lambda),$$

$$\eta(r - \lambda) = -\dot{\eta} + \eta(\rho - \lambda),$$

where $P_C \equiv [\int_0^N p_j^{1-\epsilon} dj]^{\frac{1}{1-\epsilon}}$. Apparently, the first two equations are the optimal conditions for consumption and leisure, respectively, and the latter three equations are the optimal conditions for three distinct types of assets. Furthermore, a simple two-stage budgeting procedure yields the demand for the consumption good j:

$$c_j = E[\frac{p_j^{-\varepsilon}}{\int_0^N p_k^{1-\varepsilon} dk}]$$

As a result, the total demand for all goods is given by:

$$X_j = Lc_j = LE\,[\frac{p_j^{-\epsilon}}{\int_0^N p_k^{1-\epsilon} dk}]. \tag{6}$$

Accordingly, we can have the market share of firm j as follows:

$$\kappa_j = \frac{p_j^{1-\epsilon}}{\int_0^N p_k^{1-\epsilon} dk} = \frac{P_j X_j}{LE}. \tag{7}$$

Since there is a continuum of goods and each firm is atomistic, taking X_j as given, monopolistic competition then prevails and individual firms face isoelastic demand curves.

2.2 Firms

The interaction between incumbents and entrants is the core of the model. There are two dimensions of technology change in this sector – production cost reduction (vertical dimension) and variety expansion (horizontal dimension). In the vertical dimension, incumbents engage in in-house R&D in order to reduce the production costs and earn higher profits.[9] In the horizontal dimension, entrepreneurs make entry decisions and compete with incumbents for market share. Through firm entry, the number of firms N and the individual firm market share κ_j are endogenously determined. In this section, we first focus on the determination of the price and investment in R&D of incumbents given the existing market structure and then turn to the endogeneity of the market structure, which is related to the entry decisions of entrepreneurs.

2.2.1 Incumbents

The goods sector comprises a continuum of monopolistically competitive incumbents, each of which produces a single intermediate good X_j with the following technology:

$$L_{X_j} = h(Z_j)X_j, \tag{8}$$

where $h(Z_j) = Z_j^{-\theta}$, with $0 < \theta < 1$. Each incumbent j undertakes R&D to increase the knowledge Z_j. An increase in knowledge decreases the cost of production L_{X_j}. Thus, (8) can be rewritten as $X_j = Z_j^{\theta} L_{X_j}$, which indicates that an increase in knowledge improves the productivity of production labor. The firms accumulate knowledge according to:

$$\dot{Z}_j = \alpha K \, L_{Z_j}. \tag{9}$$

The flow of knowledge $\dot{Z}_j$ depends on R&D productivity α, the employment in the R&D sector of firm j, L_{Z_j}, and the stock of public knowledge:

$$K \equiv \int_0^N \kappa_j Z_j dj,$$

where κ_j is defined in (7). Note that the knowledge is non-rival within a firm and augments labor at the firm level and firm size (firm employment L_{Z_j} and L_{X_j}) is endogenously determined by firm entry. These give rise to the main difference between our model and the previous growth models without market structure, which presume that knowledge augments all labor in the economy and is not consistent with the IO findings. See Peretto (1999), Dosi (1988), Nelson

[9] Cost reducing technological progress is equivalent to quality improvement progress. See Tirole (1988).

and Winter (1992), and Malerba (1992) for the relevant discussions. As we will see later, this salient property of modeling will lead to the prediction of different results concerning the impacts of monetary policy on TFP growth.

Assume that the proportion ξ_Z of the in-house R&D investment is subject to the CIA constraint. Due to this cash constraint, incumbents have to borrow $\xi_Z L_{Z_j}$ to finance their R&D investment and return the interest rate payment to households. Accordingly, the net profit of an individual firm j can be expressed as:

$$\Pi_j = p_j X_j - L_{X_j} - (1-\xi_Z)L_{Z_j} - (1+i)\xi_Z L_{Z_j}. \tag{10}$$

The present discounted value $V_j(t)$ of net profit is given by:

$$V_j(t) = \int_t^{\infty} \Pi_j \, e^{-\int_t^{\tau} r(s)ds} d\tau. \tag{11}$$

The firm chooses the paths of its product price P_j and its R&D expenditure L_{Z_j} to maximize (11), subject to the demand function (6), production cost (8), and the R&D production function (9).

2.2.2 Entry

Entrepreneurs create new varieties to compete with incumbents for market share. To determine the entry and exit of the firm, the value of firm V_j defined by (11) has to be compared with the cost of entry and exit. For simplicity, we refer only to entry. By following Peretto (1998a), we assume that entrepreneurs have to pay a sunk cost of $\frac{1}{\beta}$ units of labor hours in order to enter the market. In the presence of the cash constraint, they have to borrow money to finance the ξ_N proportion of entry cost, i.e., the $\xi_N \frac{1}{\beta}$ units of labor hours. Therefore, the total entry cost measured in terms of labor hours is:

$$(1-\xi_N)\frac{1}{\beta} + (1+i)\xi_N \frac{1}{\beta} = (1+\xi_N i)\frac{1}{\beta}. \tag{12}$$

The free entry condition requires the value of the firm to be equal to the entry cost. That is,

$$V_j = (1+\xi_N i)\frac{1}{\beta}. \tag{13}$$

By combining the labor requirement for entry $L_N = V_j \dot{N}$ with (13), we further have

$$\dot{N} = \frac{\beta}{1+\xi_N i} L_N. \tag{14}$$

From (10) and (13), we can define the one-period loan B for the firms' R&D activities (which appears in the household budget constraint (2)) as $B = \frac{\xi_Z L_Z + \xi_N/(1+\xi_N i) L_N}{lL}$. Hall and Lerner (2010) report that in practice more than 50 percent of R&D spending is wage payments to highly skilled technology workers (scientists and engineers) and the R&D-intensive firms need to hold cash to smooth their R&D spending over time. Our specification exactly captures their observation.

2.3 Monetary Authority

The monetary authority implements a nominal interest rate peg by targeting the nominal level of the interest rate i. Let the growth rate of the nominal money supply be $\mu = \frac{\dot{M}_L}{M_L}$. Thus, by recalling that $M_m = \frac{M_L}{P_m L}$, the evolution of money real balances is: $\frac{\dot{M}_m}{M_m} = \mu - \pi - \lambda$. The monetary authority will endogenously adjust the money growth rate μ to whatever level is needed for the targeted interest rate i to prevail.

To balance its budget, the government (solely represented by the monetary authority) simply returns the seigniorage revenues to households as a lump-sum transfer T. Thus, the government budget constraint is given by:

$$T = \frac{\dot{M}_m}{P_m L} = \mu M_m = \dot{M}_m + (\pi + \lambda) M_m. \tag{15}$$

2.4 General Equilibrium

Households choose $\{C_t, l_t, B_t, M_{mt}\}$ to maximize utility (1), subject to (2) and (3), given $\{r_t, w_t\}$ and policy $\{i_t\}$. The first-order conditions of the household's maximization problem reported in Section 2.1, can be summarized as follows:

$$i = r + \pi, \tag{16}$$

$$l = 1 - \gamma E(1 + \xi_c i), \tag{17}$$

$$\frac{\dot{E}}{E} = r - \rho. \tag{18}$$

The no-arbitrage condition between assets and money (including the loans for R&D) (16) implies the Fisher equation. Equation (17) refers to a trade-off between labor supply and consumption expenditure. Equation (18) is the standard Euler equation of consumption.

Incumbents choose $\{p_t(j), L_{Zt}(j)\}$ to maximize the present value of profits (11), subject to (6) and (9), given policy $\{i_t\}$. Entrants make entry decisions, given $\{V_t(j)\}$, entry cost (12) and policy $\{i_t\}$. By following Peretto (1998a), it is easy to prove that under certain parameter restrictions, all firms make symmetric decisions. Accordingly,

Proposition 1. *Assuming $\theta(\epsilon - 1) < 1$, the Nash Equilibrium is symmetric, under which the goods prices, returns to in-house R&D, and returns to entry, respectively, are:*

$$p = h(Z)\frac{\epsilon}{\epsilon - 1}, \tag{19}$$

$$r_Z = \frac{\alpha}{1 + \xi_Z i}[\frac{\theta(\epsilon - 1)LE}{\epsilon N} - (1 + \xi_Z i)\frac{L_Z}{N}], \tag{20}$$

$$r_N = \frac{\pi}{V} + \frac{\dot{V}}{V} = \frac{\beta}{1 + \xi_N i}[\frac{LE}{\epsilon N} - (1 + \xi_Z i)\frac{L_Z}{N}], \tag{21}$$

where L_Z is the aggregate employment in the R&D sector.

Proof All proofs are relegated to the Appendix. ■

Recall that θ measures the degree of diminishing returns of R&D to production and ϵ is the elasticity of substitution of intermediates. Thus, the condition $\theta(\epsilon - 1) < 1$ guarantees that the diminishing return to R&D is high enough so that no firms have the incentive to engage in more R&D than others (see Peretto (1998b) for the details). With this condition, Proposition 1 shows that the returns to R&D positively depend on firm market size, which is the total expenditure LE times the market share $1/N$. In the traditional growth model without market structure, the return to R&D depends only on LE. Any policy that alters LE will affect the returns to R&D and therefore the balanced growth rate. This implies a scale effect. In our model, the market share $1/N$ is endogenously determined by the firm's entry decision. Thus, a policy which alters LE will also lead to an adjustment of N accordingly, while leaving LE/N and $r's$ unchanged. As is evident, the endogeneity of the market structure in this model allows for the discussion on how a policy affects the market structure (N) and therefore LE/N, such that the returns to R&D and the growth rate are affected, which cannot be done using the other types of models.

The model generates three different growth regimes: the regime with only in-house R&D, the regime with only firm entry, and the regime with both. We focus on the regime with both in-house R&D and firm entry. Therefore, following Peretto (1998a), we impose the following parameter restrictions:

$$\frac{\alpha}{1+\xi_Z i} > \frac{\alpha\theta(\epsilon-1)}{1+\xi_Z i} > \frac{\beta}{1+\xi_N i}. \tag{22}$$

To ensure the market-clearing condition of the goods market, the total supply of goods measured by labor cost is equal to the total demand measured by household expenditure, i.e.,

$$L_X = NL_{X_i} = \frac{\epsilon-1}{\epsilon}LE. \tag{23}$$

In addition, the market-clearing condition of the financial market leads to $r = r_Z = r_N$. Accordingly, setting (20)=(21) yields:

$$L_Z = \frac{LE}{\epsilon}\frac{\left[\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}\right]}{\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta}. \tag{24}$$

Finally, the labor market clears implying that

$$Ll = L_N + L_Z + L_X, \tag{25}$$

where $L_Z = \int_0^N L_{Zj}dj = NL_{Zj}$, $L_X = \int_0^N L_{Xj}dj = NL_{Xj}$, and l is reported in (17).

3 Monetary Policy and Economic Growth

In this section, we solve the dynamic system and then analyze both the steady-state and transition effects of an increase in the nominal interest rate. Define firm size as $s = \frac{L}{N}$, effective firm size as $s_f = \frac{Ll}{N}$, the TFP growth as $g = \theta\frac{\dot{Z}}{Z}$, and hence the consumption growth as $g_c = \frac{\epsilon}{\epsilon-1}\frac{\dot{N}}{N} + \frac{\dot{c}_i}{c_i}$. Combining (24) and (9), we have

$$g = \alpha\theta\frac{LE}{\epsilon N}\frac{\left[\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}\right]}{\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta}. \tag{26}$$

It is clear from the above equation that with an endogenously-determined N, the TFP growth depends on $\frac{LE}{N}$ which is the firm's market size, rather than the aggregate market size LE. This scale-invariant property is an important difference from the previous models without market structure in which the TPF growth depends on the aggregate expenditure.

Given (26), by using the optimal labor supply (17), labor requirement for production (23), labor clearing condition (25), free entry condition (14), no-arbitrage condition that (21)=(20), and Euler equation (18), we can reduce the whole dynamic system to the following two differential equations in terms of g and s:

$$\frac{\dot{s}}{s} = \lambda - \frac{\beta}{1+\xi_N i}(s - \frac{g}{\alpha\theta}\Omega), \tag{27}$$

$$\frac{\dot{g}}{g} = \frac{g}{\alpha\theta}\frac{\beta}{1+\xi_N i}\{\frac{\alpha[1-\theta(\epsilon-1)]}{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}} + \Omega\} - \rho + \lambda - \frac{\beta}{1+\xi_N i}s, \tag{28}$$

where $\Omega = \frac{\gamma(1+\xi_c i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}} + \frac{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_Z i} + (\epsilon-1)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}}$. The loci of the system are given by:

$$\dot{s} = 0 \Rightarrow g = Q_1 \cdot (s - \frac{1+\xi_N i}{\beta}), \tag{29}$$

$$\dot{g} = 0 \Rightarrow g = Q_2[s + \frac{1+\xi_N i}{\beta}(\rho - \lambda)]. \tag{30}$$

Note that Q_1 and Q_2 are complicated functions of i and $\xi_q, q = (Z, N, C)$, which are relegated to the Appendix. From (29) and (30), the phase diagram can be expressed in Figure 1.

3.1 Steady-State Effects

In the steady state, the TFP growth rate $g^\star$ and the ratio of the labor force to the number of firms $s^\star$ are solved by setting $\dot{s} = 0$ and $\dot{g} = 0$. Given these, the steady-state inflation rate $\pi^\star$ is determined by (16) and (18) with $\dot{E} = 0$, while the consumption expenditure per capita $E^\star$ is determined by the market-clearing condition of the financial market (20)=(21) with (9). With the steady-state $s^\star$ and $E^\star$, we can use (17) to pin down the steady-state employment rate $l^\star$ and in turn the effective firm size $s_f^\star$. Finally, given that $g_c = \frac{\epsilon}{\epsilon-1}\frac{\dot{N}}{N} + \frac{\dot{c}_i}{c_i}$, the steady-state growth rates of entry $(\frac{\dot{N}}{N})^\star$ and consumption $g_C^\star$ can further be determined by using (14), (23), (24), and (25). See the Appendix for the detailed deduction.

All results are summarized in Proposition 2.

Proposition 2. *There is a nondegenerate, competitive equilibrium of growth, which is stable and unique. On the growth path,*

$$g^\star = \theta\rho\frac{1}{\beta}[\frac{\alpha\frac{1+\xi_N i}{1+\xi_Z i}\theta(\epsilon-1) - \beta}{1-\theta(\epsilon-1)}], \tag{31}$$

$$s^\star \equiv (\frac{L}{N})^\star = \frac{1+\xi_N i}{\beta}\{\rho\frac{\gamma(1+\xi_c i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + \frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\epsilon_N i} + (\epsilon-1)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\alpha[1-\theta(\epsilon-1)]} + \lambda\}, \tag{32}$$

$$l^\star = \frac{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i} + (\epsilon-1)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + \alpha[1-\theta(\epsilon-1)]\frac{\lambda}{\rho}}{\gamma(1+\xi_c i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + \frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\epsilon_N i} + (\epsilon-1)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + \alpha[1-\theta(\epsilon-1)]\frac{\lambda}{\rho}}, \tag{33}$$

$$s_f^\star \equiv (\frac{Ll}{N})^\star = \frac{1+\xi_N i}{\beta}\rho\{\frac{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i} + (\epsilon-1)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\alpha[1-\theta(\epsilon-1)]} + \frac{\lambda}{\rho}\}, \tag{34}$$

$$(\frac{\dot{N}}{N})^\star = \lambda, \tag{35}$$

$$g_C^\star = \frac{\dot{C}}{C} = \frac{\epsilon}{\epsilon-1}\lambda + \theta\rho\frac{1+\xi_N i}{\beta}[\frac{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}}{1-\theta(\epsilon-1)}]. \tag{36}$$

$$E^\star = \frac{\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\gamma(1+\xi_c i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + \frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\epsilon_N i} + (\epsilon-1)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + \alpha[1-\theta(\epsilon-1)]\frac{\lambda}{\rho}} \tag{37}$$

$$\pi^\star = i^\star - \rho \tag{38}$$

As shown in Proposition 2, the TFP growth $g^\star$ does not depend on the aggregate population L, due to the scale-invariant property of the model. Most notably, $g^\star$ positively depends on in-house R&D productivity α and negatively on entry productivity β. The reasons are very intuitive. A higher in-house R&D productivity induces firms to engage in more in-house R&D and hence increases TFP growth. A higher entry productivity means that entry is easy so that more firms enter the market, which leads to a smaller firm market size and less return on in-house R&D. Therefore, firms engage in less in-house R&D and the TFP growth is lower.

We are ready to investigate how an increase in the nominal interest rate i affects the long-run economic performance through various CIA constraints. Based on Proposition 2, we use Figures 2 and 3 to present the results of the comparative statics and establish the following two corollaries.

Corollary 1. *With a CIA constraint on in-house R&D ($\xi_Z > 0$, $\xi_c = \xi_N = 0$), a higher nominal interest rate i decreases the steady-state effective firm size, TFP growth rate and consumption growth rate, while it increases the inflation rate. In addition, it has an ambiguous effect on employment and consumption expenditure per capita.*

As predicted by the Fisher equation, a higher nominal interest rate i increases the long-run inflation rate shown in (38). A higher inflation rate raises the cost of holding money and hence reduces real money balances in the economy. If the money balances are required to engaging in in-house R&D and entry investment is not restricted by such a constraint, in-house R&D becomes more expensive compared to firm entry. A higher i makes both r_z and r_N decrease, but r_Z decreases more than r_N, i.e., $r_Z < r_N$ by referring to (21) and (20). Therefore, the economic (labor) resource shifts away from the quality improvement-type to the variety expansion-type innovations, which restores the equilibrium such that $r_N = r_Z$ holds again. This implies that the number of firms expands faster than the population ($\dot{N}/N > \lambda$) and the firm size (L/N) thereby shrinks. In the steady state, small-sized firms engage in less in-house R&D, which decreases the rate of innovation growth shown in (9). Since the consumption growth g_C, as indicated in (36), is a weighted sum of the TFP growth g and the growth rate of entry $\dot{N}/N$ (which is equal to λ in the steady state), the consumption growth rate decreases as well. This negative growth effect is similar to that of Chu and Cozzi (2013).

In addition, the response of equilibrium employment could be negative or positive. In the steady state, the entry rate $\dot{N}/N$ is pinned down by the growth rate of population λ, and therefore, L_N is unresponsive to the monetary policy change, as shown in (14). However, a higher i restricts in-house innovations, leading labor resources to shift away from R&D (a decrease in L_Z) to production (an increase in L_X). Due to these two conflicting effects of resource reallocation, the equilibrium employment may be either increasing or decreasing in the nominal interest rate, as indicated in (25). Moreover, it is clear from (17) that there is a trade-off between consumption expenditure and labor supply. Thus, the steady-state consumption expenditure changes in the opposite direction of employment and has an uncertain response to a change in monetary policy. Interestingly, it is shown that a higher nominal interest rate may increase the consumption expenditure per capita, even though it gives rise to a negative effect on the long-run consumption growth rate. When a higher i reduces the TFP growth, less R&D makes the (quality-adjusted) price of goods decrease more slowly than in the case without the policy change. Therefore, households consume less, but may incur higher expenditure.

Corollary 2. *With a CIA constraint on firm entry investment ($\xi_N > 0$, $\xi_c = \xi_Z = 0$), a higher nominal interest rate i increases the steady-state effective firm size, inflation, employment, and* *the growth rates of TFP and consumption, but decreases the consumption expenditure per capita.*

If entry investment is subject to the CIA constraint, raising the nominal interest rate decreases the real money balances in the economy, owing to a rise in inflation, which restricts the variety-expanded innovations. This decreases r_N, keeping r_Z unchanged, as shown in (21) and (20). Because $r_N < r_Z$, the resource shifts from in-house R&D to entry investment, which leads to $\dot{N}/N < \lambda$ and increases firm size. The expansion in firm size further leads to more in-house R&D, resulting in higher growth rates of TFP and of consumption expenditure. In a Schumpeterian model, to gain a higher growth rate, some degree of monopoly power is needed to act as the reward accruing to the successful firms from their innovations. Based on this logic, our monetary model suggests that if entry investment is subject to a higher degree of cash constraint, a rise in the nominal interest rate renders the existing innovators with a larger degree of monopoly power by increasing entry costs to potential competitors. Thus, high inflation can be associated with higher growth and the Mundell-Tobin effect is valid. Such a case could be empirically plausible, because the evidence shows that R&D is more likely to be liquidity constrained for young entrants.

Moreover, the equilibrium employment increases, since the labor demand of both in-house R&D and entry increases in the steady state. It is notable that even though the entry growth does not change ($\dot{N}/N = \lambda$ in the steady state), the labor requirement for entry L_N becomes higher, because a higher i decreases the productivity of entry, as shown in (14). In addition, as mentioned above, the consumption expenditure per capita changes in the opposite direction of employment and hence decreases in the steady state. It turns out that while consumption expenditure decreases, the consumption growth rate increases. When entry is restricted by the CIA constraint, a higher i renders the incumbents with an effective shield against potential competition, which motivates them to engage in more in-house R&D and in turn raises the TFP growth. In the meantime, more R&D decreases the price of (quality-adjusted) goods and therefore households can enjoy more consumption by incurring less expenditure.

For ease of comparison between the two cases, we further summarize the comparative statics above in the following table:

	$g^\star$	$s^\star$	$g_c^\star$	$l^\star$	$s_f^\star$	$(\frac{\dot{N}}{N})^\star$	π	$E^\star$
CIA constraint on in-house R&D	↓	↓	↓	↑↓	↓	–	↑	↑↓
CIA constraint on Entry	↑	↑	↑	↑	↑	–	↑	↓

The table shows that a higher nominal interest rate is associated with a higher inflation rate, regardless of whether in-house R&D or entry is restricted by money balances. However, a higher nominal interest rate can increase or decrease economic growth, depending on the two distinct cash constraints. That is, our model gives rise to a mixed long-run relationship between growth and inflation. This is consistent with recent empirical evidence; Bullard and Keating (1995), Bruno and Easterly (1998), and Ahmed and Rogers (2000) find that the real output/growth effect of inflation is largely insignificant except in high-inflation countries.

How does the government's monetary policy affect the market structure? It is clear from the above table that the various CIA constraints end up with very different market structures. Targeting a higher nominal interest rate is unfavorable to the variety expansion-type R&D, if entry investment is subject to a relatively high cash constraint. Under such a situation, the market is characterized by a small number of large-sized firms. By contrast, a higher i is unfavorable to the quality improvement-type R&D, if in-house R&D is restricted by a larger cash constraint. As a result, the market is characterized by a large number of small-sized firms. A testable hypothesis is to empirically examine the relationship between the market concentration and monetary policy. Such a test is important, because it is well-known that the industrial distribution of firm size is crucial to an economy's performance in terms of growth (see, e.g., Pagano and Schivardi 2003) and that the distinct firm sizes give rise to quite different responses to business cycles (see, e.g., Bernanke et al. 1996). In addition to the conventional anti-trust policy and regulatory reform, the government should learn how to use monetary policy to govern the market structure in an R&D-intensive economy.

In a traditional CIA growth model with flexible labor (e.g., Gomme 1993 and Wang and Yip 1992), there is a negative effect of inflation on economic growth in a Lucasian economy in which real money balances are required prior to purchasing the consumption good. However, in a second-generation growth model, Chu and Ji (2012) find that growth is immune to monetary policy, while the equilibrium employment is responsive. Since our model is free from the scale effect, Proposition 2 can easily recover their result by setting $\xi_c > 0$ and $\xi_Z = \xi_N = 0$. A more interesting note is that raising i decreases the steady-state consumption expenditure without changing the consumption growth rate, because the market size per firm EL/N is unresponsive to the monetary policy in the steady state. Accordingly, our model indicates that monetary policy may only have a level effect on consumption expenditure, but no long-run growth effect on consumption, when only consumption is subject to the CIA constraint.

3.2 Transition Effects

As emphasized by Peretto (1998a), to make the growth model of market structure shining, it is important to further examine the transition effects, to which we now turn. From (29) and (30), we have:

$$\frac{\partial Q_1}{\partial i} = [-\frac{\partial(\frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\partial i}A - B] \cdot D_1^2; \quad (39)$$

$$\frac{\partial Q_2}{\partial i} = [-\frac{\partial(\frac{1+\xi_Z i}{1+\xi_N i}\beta)}{\partial i}(A + A^{'}) - (B + B^{'})] \cdot D_2^2; \quad (40)$$

where A, B, $A^{'}$, $B^{'}$,D_1 and D_2 are all complicated combinations of parameters, whose exact expressions are relegated to the Appendix. As shown in the Appendix, they are all positive and $D_1 > D_2$. We can also easily see that $\frac{\partial Q_1}{\partial i} < 0$, $\frac{\partial Q_2}{\partial i} < 0$ under the condition of $\xi_c = \xi_N = 0$, while $\frac{\partial Q_1}{\partial i} > 0$, $\frac{\partial Q_2}{\partial i} > 0$ under the condition of $\xi_c = \xi_Z = 0$. In either case, $|\frac{\partial Q_1}{\partial i}| < |\frac{\partial Q_2}{\partial i}|$ holds true. These indicate that in response to a rise in i both the $\dot{g} = 0$ and $\dot{s} = 0$ loci shift downwards with the former shifting more than the latter, if the R&D investment is subject to the cash constraint. By contrast, both the $\dot{g} = 0$ and $\dot{s} = 0$ loci shift upwards with the former shifting more, if the entry investment is subject to the cash constraint. As shown in Figures 2 and 3 (Corollaries 1 and 2), a higher nominal interest rate decreases both the steady-state growth rate and firm size as the R&D investment is subject to the cash constraint, but it increases both of them as the entry investment is subject to the cash constraint.

CIA constraint on in-house R&D

An increase in i gives rise to a wedge between the returns to R&D and to entry. It is favorable to entry, i.e., $r_Z < r_N$, if the in-house R&D is restricted by the CIA constraint. Given a predetermined N, economic resources shift out from in-house R&D to entry, leading TFP growth g to jump down on impact (referring to (9)) and the entry rate $\dot{N}/N$ to jump up (referring to (14)), as shown in Figure 4. Since the number of firms expands faster than the population, the firm size (s) decreases along the transitional path. Given that small-sized firms engage in less R&D, this implies that TFP growth g gradually declines to a lower steady-state rate until the growth rate of population rate returns to the steady-state value λ.

As noted previously, the consumption growth rate is a combination of TFP and population growth. As a result, the growth rate of consumption may jump up or down on impact, since the population growth rate jumps up, while the TFP growth rate initially jumps down. Afterwards, the consumption growth rate gradually converges to a lower value of the steady state, given that the TFP growth and population growth both gradually decline in transition.

Corollary 1 indicates that a higher nominal interest rate i shifts the labor resource away from R&D (L_Z) to production (L_X), giving rise to a mixed effect on the equilibrium employment rate l. This resource reallocation effect governs employment not only in the long-run steady state, but also in the short-run transition. Thus, as shown in Figure 4, on impact, employment could either jump down or up, and afterwards it monotonically converges to a higher (lower) steady state, if the resource reallocation effect is more (less) favorable to the demand for labor in the production sector. With regard to the transition of consumption expenditure, (17) demonstrates that its trajectory is opposite to that of employment.

CIA constraint on entry

If entry, instead of R&D, is subject to the CIA constraint, a higher i leads the entry investment to become more expensive, relative to the in-house R&D. When the resources move away from entry to in-house R&D, Figure 5 shows that g jumps up, but $\dot{N}/N$ jumps down at the moment of the policy change. As $\dot{N}/N$ grows more slowly than the population λ, the firm size s goes up, leading to higher values of r_Z and r_N. Therefore, on the one hand, the TFP growth goes up further and gradually converges to a new and higher steady-state value. On the other hand, the entry growth also gradually increases until it returns to the steady state λ.

As a result of the adjustments of g and $\dot{N}/N$, the consumption growth rate, with a jump on impact, gradually increases to a higher steady-state value. Of particular interest, because more intensive R&D activities decrease the prices of products, households can increase their consumption but incurring less expenditure. That is why the consumption expenditure E exhibits a transitional trajectory, which is just the opposite of that of the consumption growth rate, as shown in Figure 5.

Based on the above analysis of transition, we establish the following proposition.

Proposition 3. *In response to an increase in the nominal interest rate i, the transitional adjustment of the firm size s and the TFP growth g are monotone: both monotonically decrease (resp. increase) to the steady-state value, if in-house R&D (resp. entry) is subject to the CIA constraint. In either case, along the transition path the consumption growth rate g_C, the employment rate l, and the consumption expenditure E may mis-adjust from their long-run steady states.*

We have so far investigated the case with the cash constraint on either in-house R&D or entry separately. An extended discussion which allows the coexistence of all cash constraints (including a cash constraint on consumption) may be of interest and worth noting.

Remark

In absence of the CIA constraint on consumption, our analysis shows that the transitional adjustment of economic growth is monotone in the case with the cash constraint on either in-house R&D or entry. Moreover, it is easy to find that if both in-house R&D and entry investment coexist and are subject to the same degree of cash constraint ($\xi_N = \xi_Z > 0$), the two conflicting effects cancel each other out and, as a result, monetary policy has no effect on the TFP growth. Is it possible for the TFP growth rate to exhibit an interesting mis-adjustment (in the sense that along the *transition* path g mis-adjusts from its *long-run steady state*)? This is an interesting and possibly more realistic case. To this end, we consider the situation where $\xi_c > 0$ and $\xi_N > \xi_Z > 0$. The condition $\xi_N > \xi_Z > 0$ captures the IO fact that newer firms face a larger cash constraint than older firms. This allows us to generate a positive growth effect of inflation in the long run, as predicted by the Mundell-Tobin effect. The condition $\xi_c > 0$, on the one hand, satisfies the common specification of a CIA model, and on the other hand, enables us to capture the idea of Chu and Ji (2012). If consumption is restricted by a cash constraint, raising the nominal interest rate decreases the real money balances, which in turn lower consumption and employment. If the CIA constraint parameter on consumption ξ_c is significantly high, then the decrease in employment leads the growth rate to jump down in transition.[10] Thus, as shown in Figure 6, in response to an increase in i, the TFP growth rate rises in the long run, while it falls during the transition.

4 Conclusion

This paper has explored the long-run steady-state and the short-run transition effects of monetary policy on the number of firms, firm market size, inflation and economic growth. In the study, a Schumpeterian growth model with an endogenous market structure has been constructed and CIA constraints have been imposed on two types of R&D investment - in-house R&D (quality-improved R&D) and entry investment (variety-expanded R&D). This is the first time a second-generation growth model has been used to study the relationship between monetary policy and economic growth through a CIA constraint on distinct types of R&D investment and such a model is consistent with various IO, growth, and monetary economic facts.

Our comparative statics analysis has shown that if in-house R&D is subject to the CIA constraint, raising the nominal interest rate increases the market size, but decreases the firm size and economic growth. In sharp contrast, if entry investment is subject to the CIA constraint, a higher nominal interest rate has opposite effects on these variables. In either case, inflation positively responds to such a monetary policy. These results have provided a couple of new implications of relevance to policy. First, we have identified a new channel for the Mundell-Tobin effect. Second, by shedding light on the variety of CIA constraints, the mixed long-run

[10]See the Appendix for the threshold of ξ_c.

relationship between growth and inflation can reconcile the recent empirical findings. Third, in the presence of various CIA constraints on R&D, the identical monetary policy may end up with very different market structures. This prediction has provided an empirically testable hypothesis concerning the relationship between market concentration and monetary policy.

Our transition analysis has indicated that in response to a higher targeting level of the nominal interest rate, firm size and TFP growth both monotonically decrease (resp. increase) to the steady-state value, if in-house R&D (resp. entry) is subject to the CIA constraint. However, in either case, along the transition path the consumption growth rate, the employment rate, and the consumption expenditure may mis-adjust from their long-run steady states. In particular, when all cash constraints on R&D and consumption are allowed to play a role, the TFP growth rate may also exhibit a mis-adjustment.

As a future agenda, it would be interesting to ask how the monetary policy affects social welfare. Is the Friedman (1969) rule socially optimal in the growth model with an endogenous market structure? Roughly speaking, with a CIA constraint on R&D, a higher i may have an ambiguous impact on welfare. This is because the change in monetary policy makes the TPF growth g (the effect stemming from the quality-improved R&D) and the entry growth $\dot{N}/N$ (the effect stemming from the variety-expanded R&D) move in opposite directions, and thus the response of the consumption growth g_C depends on which force dominates. This ambiguity potentially implies that a positive nominal interest rate could be desirable to the society and Friedman's rule is not necessarily optimal. We must bear in mind, however, that this welfare analysis will come at the cost of much greater complexity, due to the complicated effects of the dynamic transitions. To pin down the welfare responses, we will need to further calibrate the related parameters of the model and numerically perform the analysis.

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Appendix: (A major portion of the Appendix is not intended for publication.)

Proof of Proposition 1:

For the proof of the symmetric condition $\theta(\epsilon-1)<1$ the reader can refer to Peretto's (1998b) Proposition 1. Under this condition, the incumbent chooses the paths of its product price P_j and its R&D expenditure L_{Z_j} to maximize (11) subject to the demand function (6) and the R&D production function (9). By defining q_j as the costate variable, which is the value of the marginal unit of knowledge, this optimization problem is to maximize the following current-value Hamiltonian

$$CVH_j=[p_j-h(Z_j)]X_j-(1+i)\xi_Z L_{Z_j}-(1-\xi_Z)L_{Z_j}+q_j,$$

s.t. (6) and (9). The firm's knowledge stock Z_j is the state variable, while the in-house R&D resource L_{Z_j} and the product price p_j are the control variables. By taking the first-order derivative with respect to p_j, we can obtain the optimal price, reported in (19). Moreover, the linear Hamiltonian yields

$$L_{Z_j}=\begin{cases}0 & for\, 1+\xi_Z i>q_j\alpha K\\ L_Z/N & for\, 1+\xi_Z i=q_j\alpha K\\ \infty & for\, 1+\xi_Z i<q_j\alpha K\end{cases},$$

where $1+\xi_Z i$ is the marginal cost of R&D and $q_j\alpha K$ is the value of the marginal unit of knowledge. The interior solution is determined under the condition that the marginal cost of R&D equals its marginal benefit. The differential equation for the costate variable gives:

$$r_j=\frac{\dot{q}_j}{q_j}-\frac{h'(Z_j)X_j}{q_j}, \tag{41}$$

indicating that the return to R&D is the ratio of the revenue from the innovation to its shadow price $(-h'(Z_j)X_j/q_j)$ plus the change in the value of the knowledge stock $(\dot{q}_j/q_j)$. Consider the interior solution and let $g_K=\dot{K}/K$ be the growth rate of public knowledge. Taking logs and time derivatives of $1+\xi_Z i=q_j\alpha K$, (6), (7), (8), (9), (19) and $h(Z_j)=Z_j^{-\theta}$ allow us to reduce (41) to (20) under the symmetric equilibrium.

Given entry costs $(1+\xi_N i)\frac{1}{\beta}$ and the value produced V_j, taking logs and time derivatives of the free entry condition (13) yields:

$$r_j = \frac{\pi_j}{V_j} + \frac{\dot{V}_j}{V_j}. \tag{42}$$

This implies that the rate of return on the firm ownership equals the rate of return on the riskless loan of V_j. By using (13), (6), (7), (8), (19), and (10), and imposing symmetry, we can reduce (42) to (21).

Proof of Proposition 2:

From (29) and (30), it is easy to derive the steady state values of g and s, as reported in (31) and (32). From the arbitrage condition (21)=(20), we obtain (24). By recalling that $g = \theta \frac{\dot{Z}}{Z}$ and $\dot{Z} = \alpha K \frac{L_Z}{N}$, we then have $g = \theta\alpha\frac{L_Z}{N}$ under the symmetry (i.e., $Z_j = K$). Using (31), (32) and (24), one can solve the steady state value of E through solving $g = \theta\alpha\frac{L_Z}{N}$. By plugging the steady state value of E into the optimal labor supply (17), the steady state value of l can be derived, as shown in (33). The steady state value of s_f, reported in (34), is the product of (33) and (32). The steady-state entry rate (35) can be solved through $\frac{\dot{L}}{L} - \frac{\dot{N}}{N} = \frac{\dot{s}}{s}$, given that $\frac{\dot{L}}{L} = \lambda$ and $\frac{\dot{s}}{s} = 0$ in the steady state.

From (4), we have:

$$g_C = \frac{\epsilon}{1-\epsilon}\frac{\dot{N}}{N} + \frac{\dot{c}_j}{c_j}.$$

From (6), (8), (23), and (18) and by imposing the symmetric condition $L_X = NL_{X_j}$, we further obtain:

$$g_C = \frac{1}{\epsilon - 1}\frac{\dot{N}}{N} + g\{1 + \frac{\beta}{(1+\xi_N i)\theta}[\frac{1-\theta(\epsilon-1)}{\frac{\alpha}{1+\xi_Z i}\theta(\epsilon-1) - \frac{\beta}{1+\xi_N i}}]\} - \rho. \tag{43}$$

The steady state value of g_C, reported in (36), is then solved by using (43), (31) and (35). Finally, the steady-state inflation rate is pinned down by the Fisher equation (16).

The Derivatives of Q_1, Q_2, A, B, A', B',D_1 and D_2:

It is easy to obtain these derivatives, which are expressed as follows:

$$Q_1 = \frac{\alpha\theta[\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta]}{\gamma(1+\xi_c i)(1+\xi_Z i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + [\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta] + (\epsilon-1)(1+\xi_Z i)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)},$$

$$Q_2 = \frac{\alpha\theta[\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta]}{\gamma(1+\xi_c i)(1+\xi_Z i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + [\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta] + (\epsilon-1)(1+\xi_Z i)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + \alpha(1+\xi_Z i)[1-\theta(\epsilon-1)]},$$

$$A = [\gamma(1+\xi_c i) + (\epsilon - 1)](1+\xi_Z i)\alpha[1-\theta(\epsilon-1)],$$

$$B = [\gamma\xi_Z(1+\xi_c i)\epsilon + (\epsilon-1)\xi_Z + \gamma(1+\xi_Z i)\xi_c\epsilon] \cdot (\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) \cdot [\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta],$$

$$A^{'} = (1+\xi_Z i)\alpha[1-\theta(\epsilon-1)],$$

$$B^{'} = \alpha\xi_Z[1-\theta(\epsilon-1)][\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta],$$

$$D_1 = \left\{\gamma(1+\xi_c i)(1+\xi_Z i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + [\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta] + (\epsilon-1)(1+\xi_Z i)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)\right\}^2,$$

$$D_2 = \left\{\gamma(1+\xi_c i)(1+\xi_Z i)\epsilon(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + [\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta] + (\epsilon-1)(1+\xi_Z i)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta) + \alpha(1+\xi_Z i)[1-\theta(\epsilon-1)]\right\}^2.$$

The Threshold of ξ_c:

If the cash constraints on in-house R&D, entry investment, and consumption exist simultaneously, in response to a higher nominal interest rate the TFP growth rate could jump down on impact, provided that ξ_c is higher than a threshold such that $\frac{\partial Q_1}{\partial i} < 0, \frac{\partial Q_2}{\partial i} < 0$, i.e.,:

$$\xi_c > \frac{(1+\xi_Z i)\alpha[1-\theta(\epsilon-1)](\gamma+\epsilon-1)\frac{\xi_N - \xi_Z}{(1+\xi_Z i)^2} - (\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)[\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta]\xi_Z(\gamma\epsilon+\epsilon-1) + A^{'} + B^{'}}{(\gamma\epsilon + 2\gamma\xi_Z i\epsilon)(\alpha - \frac{1+\xi_Z i}{1+\xi_N i}\beta)[\alpha\theta(\epsilon-1) - \frac{1+\xi_Z i}{1+\xi_N i}\beta] - \gamma(1+\xi_Z i^2)\alpha[1-\theta(\epsilon-1)]\frac{\xi_N - \xi_Z}{(1+\xi_Z i)^2}}$$

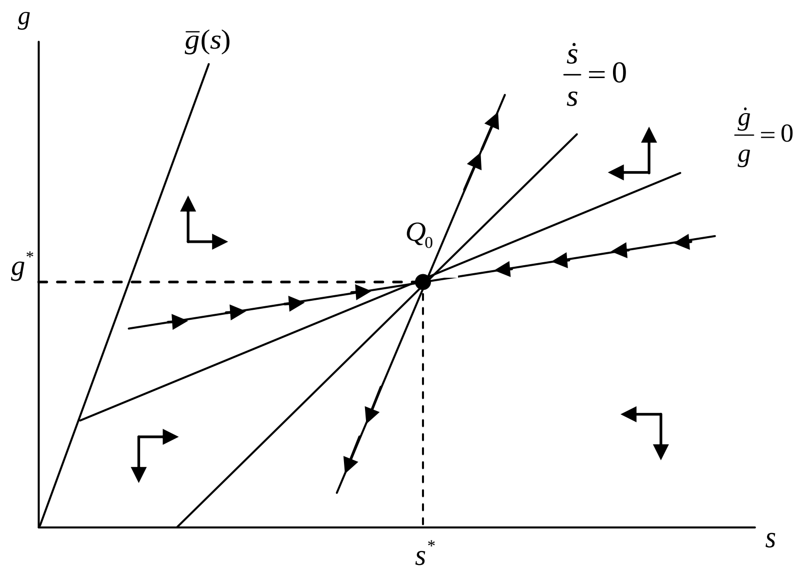

Figure 1. Phase Diagram

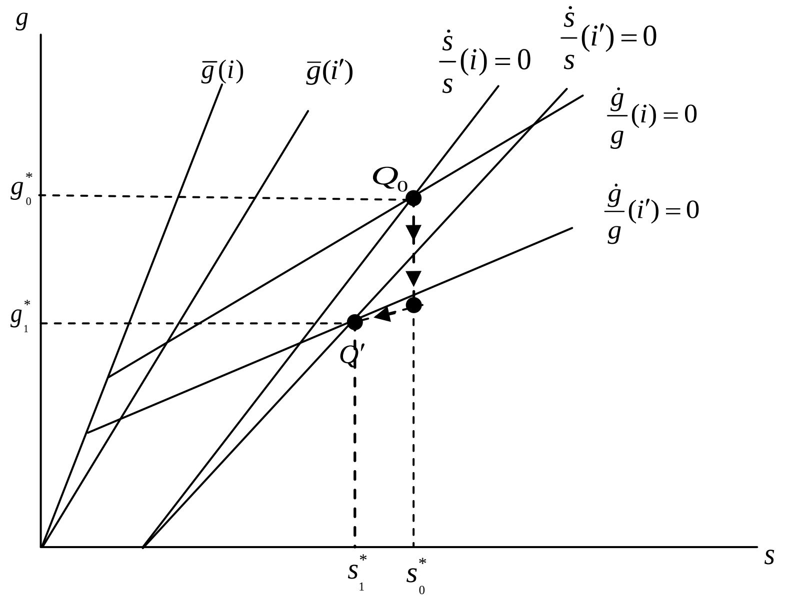

Figure 2. An Increase in i : CIA Constraint on In-House R&D

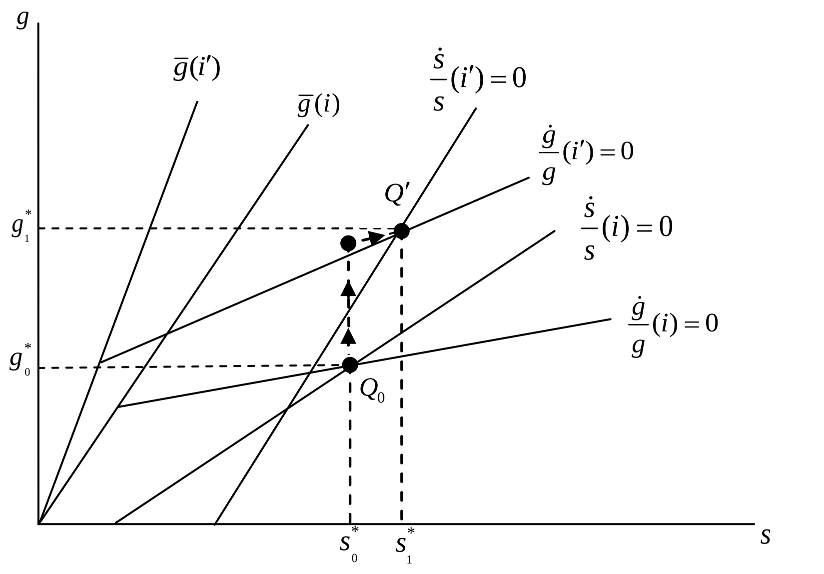

Figure 3. An Increase in i : CIA Constraint on Entry Investment

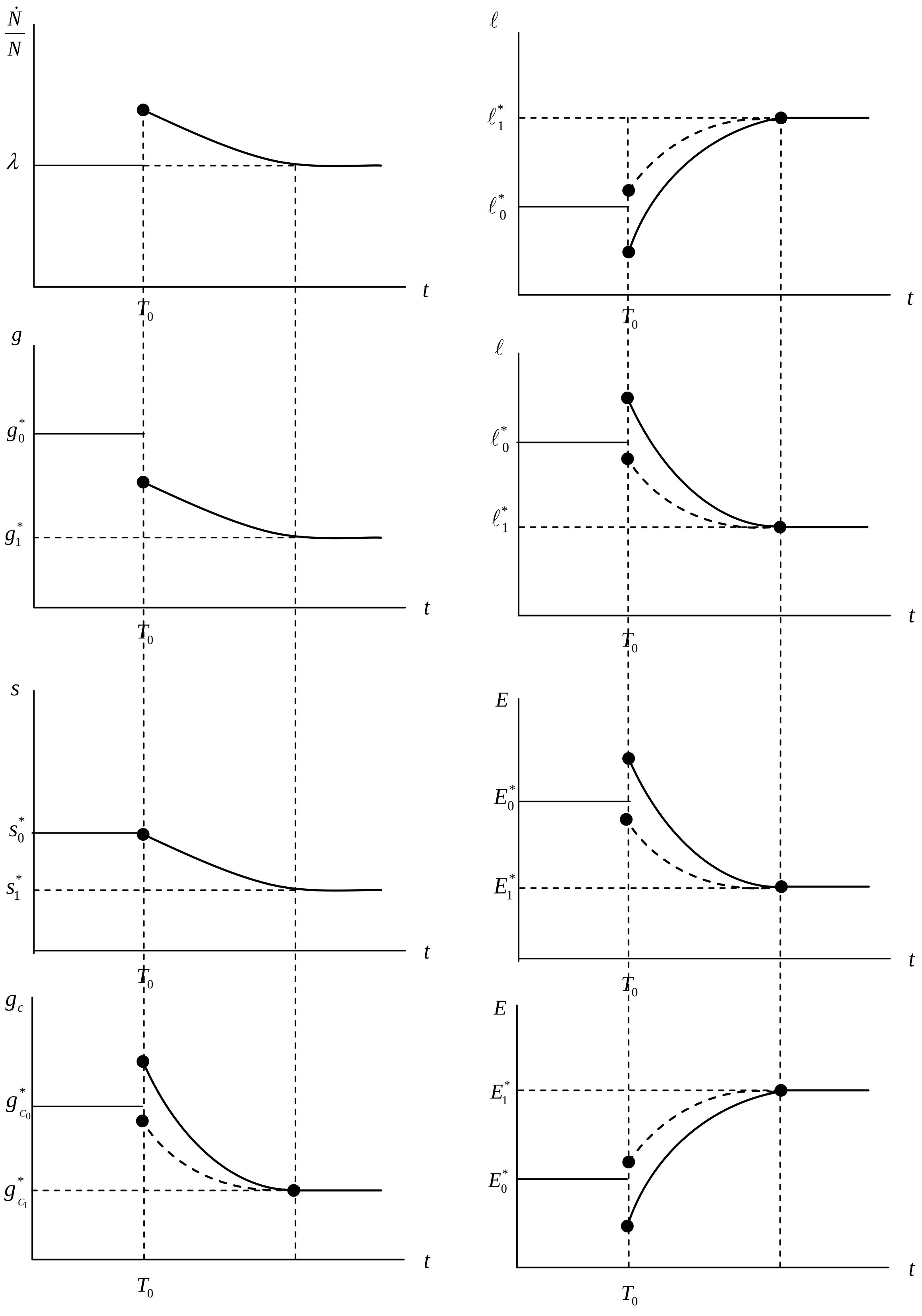

Figure 4. Time Path

An Increase in i : CIA Constraint on In-House R&D

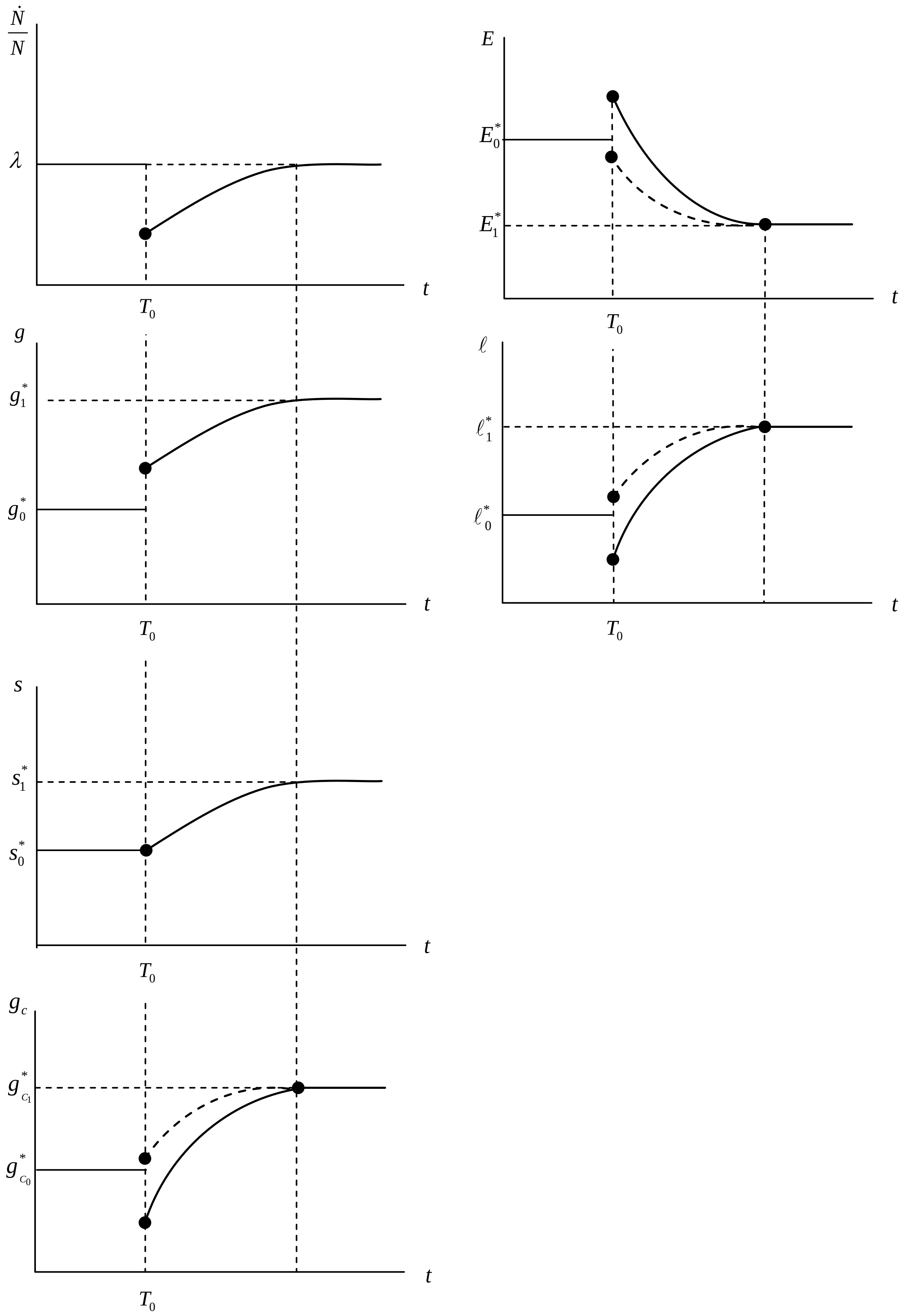

Figure 5. Time Path

An Increase in i : CIA Constraint on Entry Investment

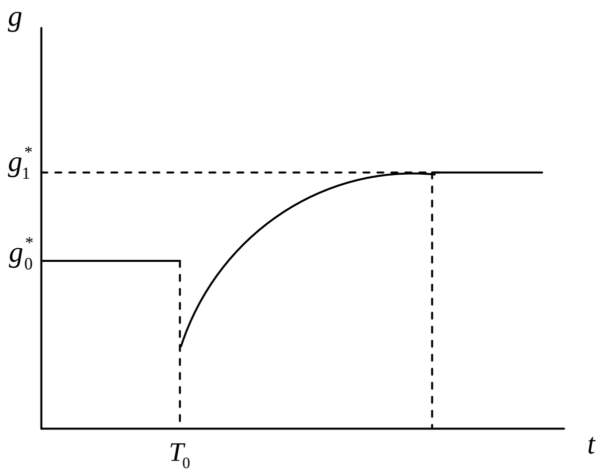

Figure 6. Time Path of Economic Growth
An Increase in i: $\xi_N > \xi_Z > 0$ and $\xi_c > 0$