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Bank Optimal Portfolio Risk Level
Under Various Regulatory Requirements

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Abstract

We investigate the impact the risk sensitive regulatory ratio may have on banks' risk taking behaviours according to two aspects: potential effects induced by the implementation of a risk sensitivity ratio and cyclical impacts that could affect risk taking behaviour. We show that the risk sensitivity of capital requirements introduce by Basel II adds either a regulatory "bonus" or "penalty" on a bank that owns a fixed capital endowment. Depending on the magnitude of cyclical variations into requirements, the "bonus" may be exploited by the bank to increase its value toward the selection of a riskier asset or the "penalty" may restrict the bank to opt for a less risky asset. Whether the optimal asset risk level swings among classes of risk through the cycle, the risk level of bank's portfolio may increase during economic upturns, or decrease in downturns, leading to a rise in financial fragility or a "fly to quality" phenomenon.

JEL Codes : G11 - G28

Keywords: Bank capital, Basel capital accord, risk incentive

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1. Introduction

Banking supervisors face the challenging task to preserve the safety of the financial system without hampering the key role banks play in the financing of the economic activity. Among the available tools to comply with these objectives, capital requirements receive important research attention. In the midst of the questions related to this dilemma, two fields emerge. On one hand, the risk taking incentive induced by regulatory requirements and its consequences on the safety of the banking system raise concerns. On the other hand, the impact of the regulatory ratio on the business cycle concentrates high research consideration. However the literature fails to reach a consensus about the effects induced by the regulatory capital to asset ratio, especially on risk taking incentives.
Concerning the impact of the capital to asset ratio on the business cycle, analyses converge toward the idea that the capital to asset ratio exacerbates the business cycle, phenomenon known as procyclicality (Blum and Hellwing 1995, Furfine 2001). Economic environment indeed affects borrowers' risk profile: as business conditions deteriorate, it becomes harder for borrowers to honour their debt and defaults may increase during recession periods. Reverse happening while economic upturns, where the improvement in economic conditions may lead to over optimist expectations on the willingness and the capability of borrowers to honour their debt. Nevertheless banks are subject to regulatory capital requirements. As bank capital, among other goals, prevents the bank from bankruptcy by absorbing borrowers' defaults, bank capital reveals as a more expensive source of funding for banks than deposits.
When recession occurs, banks may face increased borrowers' defaults that deepen their capital. To maintain their compliance with the regulatory capital to asset ratio, banks cut their loan supply, reducing the denominator of the regulatory ratio, rather than raising fresh expensive capital. This drop in loan supply affects borrowers who depend from banks to finance their activity, exacerbating therefore the recession. Conversely, when economic conditions improve, banks profits increase allowing banks to expand their loan supply by retaining part of the profits as new capital, fostering thus the expansion phase.
The procyclicality of the regulatory requirements seems to be amplified by a risk sensitive regulatory ratio (Kashyap and Stein 2004, Catarineu-Rabell et al. 2004, Heitfeild 2004, Heid 2007, Pederzolli et al. 2009). As business conditions improve, the regulatory ratio decreases to reflect the amelioration of borrowers' risk profiles, allowing an even greater expansion of banks' loan supply; whereas, during recession phases, the regulatory requirements increase as borrowers' risk profiles worsen, restricting further banks loan supply. Bank internal rating system affects borrowers risk assessment, with a point in time (PIT) rating system being more sensitive to change into current economic conditions than a through the cycle (TTC) risk assessment approach, and involves a higher procyclicality for a PIT operating banks.
Although how regulatory requirements interact with the business cycle and impact banks' lending behaviour seems to reach a consensus, researchers disagree about the magnitude and the timing of the phenomenon: how important the procyclical impact would be as how long would it last, particularly whether banks are capital constrained or display a capital buffer, remain an open debate.
The literature on that question presents the common feature to analysis the asset side of banks portfolio by segregating bonds and loans, and assumes a positive regulatory charge assigned to loans and none to bonds. Actually bonds are considered as risk-free investment, leading to zero regulatory requirement, and also as a liquid asset that can be easily sold on the market. Consequently, as economic conditions worsen, a bank facing borrowers' defaults would prefer adjusting its asset portfolio by reducing loans rather than bonds for two main reasons: only loans are computed into the regulatory ratio, and, in such recession circumstances, bank runs

may occur and push bank into insolvency if the bank cannot provide liquidity to depositors. On the contrary, when business conditions get better, banks may retain part of earnings as capital to increase its loan supply and allocate towards more loans rather than bonds, which are more profitable than the risk-free investment. The impact of a risk sensitive regulatory ratio is analysed toward the effect the business cycle induces on the requirements level on loans imposed to banks, assuming a higher level of regulatory requirements during economic downturns than during economic upturns. However, the risk level of the loan selected by the bank for its portfolio is over the field of these studies.

Zhu (2007) extends the analysis of the procyclicality with a comparative static framework where value maximizing banks have to choose their lending behaviour but also an endogenous capital target, higher than regulatory requirements when they exist, and an internal target of default probability, under various regulatory environments (no regulation, risk insensitive regulation and risk sensitive requirements). He stresses the offsetting effect of the capital buffer as the active portfolio management of banks, with dividend distribution or a rise of new capital at a linear cost, to argue that the procyclical effect of a risk sensitive ratio may be of lesser magnitude than expected in other studies. Furthermore, his analysis in term of banks dividend policy allows taking account of the next period bank capital level. He concludes to the superiority of the risk sensitive requirements from a supervisor's welfare point of view. Actually, risk sensitive capital requirements avoid distortions into the pricing of loans: as they reflect better the quality of loans, rewarding "*the high return, low risk investment opportunities*" as the diversification effect, "*the volatility is inversely related to the size of bank assets*",[1] the more accurate requirements allow for a fair pricing of loans (efficiency aspect on the banking industry). This decreases the regulatory requirements the bank faces in the current as in the next period, increasing the value of future asset earning, its franchise value, encouraging thus the bank to act prudently to preserve its future asset earnings (safety aspect).

Although the analysis conducted by Zhu (2007) focuses on the procyclical impact of regulatory requirements, but, with an asset side of banks portfolio whose composition is fixed and non adjustable, and with particular asset returns features, he builds bridges toward the lending behaviour in term of volume along the business cycle and the risk taking behaviour of bank from period to period.

Concerning the risk taking incentives that can emerge from the implementation of the capital regulation, the results provided by the literature are antagonist. Some conclude to an increase in the risk level of bank portfolio, others obtain the inverse result, i.e. a reduction of the risk level of bank portfolio.

Several views intersect. In a mean-variance framework, the enforcement of a regulatory capital to asset ratio that constrains banks reduces banks' equity value. This drop into the value of equity encourages banks to reorganize their asset portfolio toward riskier investments in order to improve their profitability and the value of bank equity capital (Koehn and Santomero 1980, Kim and Santomero 1988). Taking account of the put option related to the safety net of the deposit insurance may change this result (Keeley and Furlong 1991) but the value maximizing objective of bank shareholders involves the selection of the riskiest asset for which the put option as the bank equity values are maximum (Merton 1977). Conversely, whether the bank portfolio is initially well diversified, implementing a binding regulatory ratio reduces the risk level of the whole portfolio (Rochet 1992).[2]

1. Zhu 2007 page 4 and assumption 2 page 5.

2. Further studies focus on the adverse selection (Thakor 1996) as the monitoring efforts (Besanko and Kanatas (1996) Morrison and White 2005) the bank applies to its borrowers, with diverging conclusions, we do not present to alleviate the presentation.

To go further the static analysis, the recognition of the future earnings that constitute the franchise value leads to the intuition that a bank which displays a franchise value would act to preserve it and opt for a low risk investment strategy (Marcus 1984, Demsetz and al. 1996). However, a reduction in this franchise value, due to an increase in banking competition or to the implementation of tight regulatory requirements, may induce banks to opt for a riskier investment strategy (Blum 1999, Hellman and al 2000).
Such diverging results may be explained by the setting of each model, depending on a utility or value maximizing bank among other assumptions. Actually, Laeven and Levine (2008) provide a framework that accounts for the ownership structure of the bank capital and conducted an empirical investigation of their model. They find supports that the ownership structure (shareholders / manager) implies different reactions in term of risk taking behaviour for the same regulation: "*the relation between regulation and bank risk can actually change sign depending on ownership structure*". They also conclude that shareholders' dominated structures display a higher propensity to take risk: "*bank risk is generally higher in banks that have large owners with substantial cash-flow rights (...) although more cash-flow rights by a large owner are generally associated with greater bank risk, the importance of the large owner is weaker in economies with stronger shareholder protection laws*".[3]
Refining the dynamic analysis with a more complex setting and an endogenous bank capital target, higher than regulatory requirements, Milne and Whalley (2001) conclude that risk taking incentives depend on the level of capital buffer and regulatory requirements have no long run effect on bank risk taking. Although they do not explicitly model the asset side of the bank portfolio, in particular the distribution of asset returns that embody the future earning and hence the franchise value, they stress the critical role played by the franchise value in bank lending decisions. Banks displaying a franchise value would act to preserve it, selecting a prudent lending behaviour with a low volatility in asset earnings, whereas banks with low or without franchise value would gamble and take excessive risk, selecting a high volatility in asset earnings. However, it is notable that in their model, the bank capital target is higher for bank with low franchise value than for bank with high franchise value: high assets earnings related to an important level of franchise value provide a protection against future loose and prevent from the infringement of the regulatory requirements; low franchise value banks that want to preserve it would consequently choose a higher capital target to overcome the low earning protection. Nevertheless, one can wish more detail on the distribution of asset earnings, particularly on the volatility of such high asset earnings, to better estimate the impact on bank probability of failure, given the lower level of capital target for high franchise value than for low franchise value banks.
Milne and Whalley (2001) converges toward some of the intuition of Zhu (2007) concerning the capital buffer and the incidence the franchise value might have on the risk taking behaviour of banks from period to period, although the relationship between the level of franchise value and the desired level of capital target is non linear in their model. However, none of them explicitly model the asset side of bank portfolio and the relevant risk level of asset retained.
Moreover, the analysis conducted by Milne and Whalley (2001) shares, with others studies devoted to the risk taking behaviour, the feature to examine regulatory requirements in terms of level. These studies compare for several levels of capital requirements the reaction the bank might have regarding the risk level of the asset selected, at different degrees of formalization for the choice of the asset risk level. These comparative statics are also used to study the impact on risk taking behaviour of a risk sensitive regulation compare to a risk insensitive capital requirement.
In this paper, we follow this line of research and aim to study the impact on risk taking behaviour the regulatory requirements may have, especially the modifications in risk taking behaviour induced by the implementation of a risk sensitive ratio (Basel II) compare to a flat rate requirement

3. Laeven and Levine 2008 pages 5 and 6.

(Basel I). We also extend the analysis of business cyclicality on bank risk taking behaviour under the risk sensitive requirements to examine whether the revised regulation performs better, compare to the precedent, in fulfilling the two objectives of the banking supervision: not hampering bank lending role in the economic growth without weakening banking system safety.

We differ however from other studies presented in this introduction mainly by our assumption of invariant capital level, and an asset side of bank portfolio explicitly related to the risk level of the asset. We want to focus exclusively on the impact the regulatory ratio may have on bank risk taking behaviour, supporting this assumption of constant and non adjustable capital level. Furthermore, to concentrate the analysis on the effect of the requirements' risk sensitivity during both the transition phase as when the revised regulation is implemented, we exclude risk free investment opportunities that lead to none regulatory requirements, and model the asset side of bank portfolio with an amount set at one but with different asset risk profiles, involving different regulatory requirements for each asset risk level. To highlight the propensity to risk taking behaviour, we voluntary set a framework of bank with an ownership structure dominated by shareholders. In this line, we decide to opt for a distribution of asset returns that stresses the diverging objectives of banks (with an objective of maximizing their profitability) and supervisors (displaying the objectives of sound and efficient banking system) and we thus retain the famous "higher-risk, higher-returns" assets. Finally, our framework assumes that banks determines their own target of default probability, which is also assumed to be constant, but we model neither the choice of this internal target of default probability nor the choice of the capital endowment of the bank, we take them as given and source of heterogeneity among banks.

Although our setting exacerbates the risk taking incentives, it allows for the emergence of interesting results about the impact the risk sensitivity of the regulatory ratio may have on bank risk taking behaviour, results that cannot be excluded under less restrictive assumptions. First, the level of available capital that excess over the regulatory requirements and the initial composition of bank loan portfolio in terms of asset risk level are crucial to determine the risk taking behaviour of banks, while the shift of regulation as when the revised framework is implemented. Depending on the composition of bank loan portfolio and the level of capital buffer at the time of the switch in regulation, risk sensitive requirements can lead to a restriction or a boost in bank risk taking behaviour, as it can leave the asset risk level unchanged. Second, the cyclical impact of risk sensitive regulatory requirements in terms of asset risk level can be procyclical. On the one hand, riskier asset may be selected by the bank as business conditions improve leading to financial fragility of the banking system. On the other hand, lesser risky asset may be selected by the bank when economic conditions deteriorate leading to a "fly to quality" phenomenon.

The intuition behind these two results stands on the interaction of the regulatory risk sensitivity with the bank value maximizing objective under a panel of "higher-risk higher-returns" assets. *In fine,* bank lending decisions depends on the features of bank, its own target of default probability and its capital endowment, and bank selects the asset that fulfils its objective of value maximizing as complying with its own default probability target. According to its characteristics, bank opts for a particular asset risk level, which leads under the flat ratio to an 8 % uniform regulatory requirement. As regulation changes and becomes risk sensitive, the risk level of the asset chosen by the bank may alter regulatory requirements, leading to an increase or a decrease into regulatory charges, as it can leave the capital requirements unchanged. With the same reasoning, regulatory capital requirements vary along the business cycle, softening as economic environment improves and freeing regulatory capital, a "regulatory bonus" rewarding prudent investments as good economic states; reverse happening during recession period where regulatory requirements increase, a "regulatory penalty" that penalises risky investments as bad economic states.

What can be the reaction of a value maximizing bank which faces a "regulatory bonus"? Literature on both research fields presented above supports the idea the bank would use this bonus to expand its loan supply. We believe this bonus can either be used to increase bank value, toward the financing of a riskier asset that increases the bank value. Conversely, the "regulatory penalty" restricts the opportunity of investing into a high risk level asset. Depending on the capital buffer level, such penalty may force the bank to reduce its risk exposure to comply with its internal target of default probability, leading to the selection of a less risky asset. If the capital endowment is high enough to absorb the regulatory requirements increase and still complying with the bank own target of default probability, the bank can keep its loans portfolio unchanged.

We stress the fact that in each of these scenarios, bank complies with its internal target of default probability that remains constant. However, the risk level of bank's portfolio is altered along the business cycle into its loans composition and might increase during economic upturns, leading to a fragility of the bank and the banking system as a whole.

The rest of the paper presents the model and the solution to the optimization program of the bank with numerical simulations to illustrate the results of the transition phase into the regulation. The cyclical aspects are then studied and discussed.

2. The model

Our contribution is built on Heid (2007). We study the impact of the risk sensitivity of regulatory requirements on the choice of asset's risk level made by a bank. We also appraise the risk level of assets by the means of the asymptotic single risk factor (hereafter ASRF) model used into the regulatory requirements and consequently we make the usual assumptions, presented under a formulation closed to Repullo and Suarez (2004).

2.1. Investment projects

Consider a two periods economy with a single systematic risk factor $z \sim N(0,1)$ and a continuum of firms, indexed by i. Each firm can undertake a risky investment project, requiring one unit of wealth at period 0, and we link the firm i to its investment. Moreover, we consider that each investment project is related to a specific class of risk. We assume firms lack capital and need to borrow from a bank to undertake their project. At period 1, the investment generates a gross return equal to $(1+R_i)$ in case of project success, or $(1-\lambda)$ in case of project failure. Project success depends on the value of a random variable x_i defined by:

$$x_i = \mu_i + \sqrt{\rho}\ z + \sqrt{(1-\rho)}\ \varepsilon_i \qquad (1)$$

Where the idiosyncratic risk $\varepsilon_i \sim N(0,1)$ is independently distributed across firms and independent of the systematic risk factor z.[4] We assume μ_i is an increasing function of the specific risk of the project i while the correlation coefficient ρ reflects the sensitivity of the project to the systematic risk factor. Project i is successful only if $x_i \leq 0$, thus x_i expresses the whole level of credit risk appraised by the ASRF model.

From (1) we note that the unconditional distribution of x_i is $N(\mu_i,1)$, so the unconditional probability of default for a firm in class i is equal to:

$$\overline{p}_i = \Pr\ (\mu_i + \sqrt{\rho}\ z + \sqrt{(1-\rho)}\ \varepsilon_i > 0) = \phi\ (\mu_i) \qquad (2)$$

4. This is the usual formulation retained by the ASRF model and the IRB approach of Basel II.

Where ϕ denotes the cumulative distribution function of a standard normal random variable. The probability of default is an increasing function of the specific risk μ_i for firm i and we assume projects are such that $\overline{p}_i \in \left]0; \overline{p}_{\max}\right]$ with $\overline{p}_{\max} < 1$.

From (2) we can also determine the distribution of variable x_i conditional on the realization of the systematic risk factor z. This conditional probability of default or the default rate of project i is given by

$$p_i(z) = \Pr\left(\mu_i + \sqrt{\rho}\ z + \sqrt{(1-\rho)}\ \varepsilon_i > 0 \ \middle|\ z\right) = \phi\left(\frac{\phi^{-1}(\overline{p}_i) + \sqrt{\rho}\ z}{\sqrt{(1-\rho)}}\right) \qquad (3)$$

The default rate of a project is an increasing function of its own class of risk given by its probability of default $\overline{p}_i$, but also depends on the realization of the systematic risk factor z.

Finally, recalling that z is normally distributed, the cumulative distribution function of the default rate is given by

$$F\left(p_i(z)\right) = \phi\left(\frac{\sqrt{(1-\rho)}\phi^{-1}\left(p_i(z)\right) - \phi^{-1}(\overline{p}_i)}{\sqrt{\rho}}\right) \qquad (4)$$

The mean of the default rate distribution is the probability of default related to the class of risk of project i, while the variance is determined entirely by the exposure (sensitivity) to the systematic risk measured by ρ.

2.2. The banking system

To supply loans to firms, banks are funded with deposits D and equity capital K, and we assume that $K + D = 1$. For simplicity, we make the hypothesis of no banking intermediation costs. Bank deposits are insured through a government-funded scheme and we assume deposits receive the riskless interest rate δ. Bank capital is provided by shareholders who require an expected rate of return $(1+W)$, with $W = \delta + \omega$ and $\omega > 0$, to capture the scarcity of shareholders' wealth as the agency and/or the asymmetric information problems they face.

Banks are subject to a capital regulation and do not raise equity capital along the economic cycle: they operate most of the time with a given level of capital inherited from the previous business year, increasing their capital by retaining earnings or issuing new equities when their business plan requires. However, for a short to medium horizon of analysis, considering that banks operate with a constant capital level is a realistic assumption. As investments returns are random and regulatory infringements generate important economic costs,[5] banks make sure to operate with a higher level of capital than the regulatory requirements. The difference between the effective level of bank capital and the regulatory requirements is known as the capital buffer. It is important to note that banks actually choose their optimal investments portfolio which in turn determines the level of capital buffer.

Heid (2007) argues "*that regulatory requirements shift the bank's default point from 0* [the solvency constraint] *to a.w.* [the regulatory constraint]" and models his idea through the assumption that "*the bank sets itself a target probability of its own default*".[6] In the line of thinking of Heid, we assume banks are endowed at period 0 with a constant level of capital $\bar{K}$, which is higher than the regulatory requirements k_r, and we also retain his assumption that banks choose their own target of default probability. Moreover, banks must display a certain rating to access to the interbank market, constraint that can be materialized through

5. We do not model these costs.

6. Heid (2007) pp. 3888-3889.

this internal target of default probability $\left(1-\alpha_B^*\right)$. However, it seems to us that the intuition behind this internal target of default probability is more easily understood when it is presented under the non-defaulted perspective. Thus, α_B^* is defined as the probability of non default for bank and we use this formulation in the rest of the paper.

The bank's internal target of default probability requires that capital at period 1, i.e. equity endowment $\left(\bar{K}\right)$ and expected net returns $(\pi(z))$ generated by the investment made by the bank, must not be smaller than the regulatory requirements in α_B^* case:

$$\Pr\left[\bar{K}+\pi(z)\geq k_r\right]=\alpha_B^* \tag{5}$$

However, we model neither the choice of the optimal level of equity capital nor the selection of the optimal internal target of default probability. On the contrary, we consider them as given and source of heterogeneity among banks.[7] We rather focus on the impact the regulatory requirements have on the optimal risk level of the asset financed by the bank, given its capital endowment and its own target of default probability.

At period 0, banks decide to allocate their loans portfolio across various investment projects, picking the investment project that maximizes their value with respect to their own internal target of default probability. We assume that the rate of return on a loan financed by the bank, R_i, is an increasing function of its credit risk level, represented hereafter by the class of risk it belongs to, $\bar{p}_i$, and is measured by a spread over the riskless interest rate, denoted $S(\bar{p}_i)>0$, such that $R_i=\delta+S(\bar{p}_i)$.

At period 1, shareholders receive the net value of the bank if it is positive (no bankruptcy), and zero otherwise (assumption of limited liability). Finally, net expected returns for a bank financing a project i, conditional on the realization of the systematic risk factor z is given by:

$$\pi(z)=(1-p_i(z))(1+R_i)+p_i(z)(1-\lambda)-(1-\bar{K})(1+\delta) \tag{6}$$

The first two terms are the expected payments the bank receives in the event of the success $(1-p_i(z))(1+R_i)$ and, in the case of the failure $p_i(z)(1-\lambda)$, of the investment project and the third term represents the costs of deposits $(1-\bar{K})(1+\delta)$. As we link project to firm, and firm to bank, the net expected profits are also the net expected value of the bank at period 1. Using the above notations and simplifying (6) we obtain:

$$\pi(z)=S(\bar{p}_i)-p_i(z)\left(S(\bar{p}_i)+\delta+\lambda\right)+\bar{K}\,(1+\delta) \tag{7}$$

The objective of the bank is to maximize its expected present value at period 0, net of shareholders' initial infusion of capital, with respect to its own default probability target, that is:

$$\max_{\bar{p}_i}\; V=-\bar{K}+\frac{1}{1+W}\int_0^{\hat{p}_i(z)}\pi(z)dF(p_i(z))$$
$$s.t.\quad \Pr\left[\bar{K}+\pi(z)\geq k_r\right]=\alpha_B^*$$

Or equivalently

7. We refer the lecturer to, among others, Jeitschko and Dong Jeung (2005) for an explicit formulation of the optimal risk target chosen by the bank, and to Leland and Bjerre Toft (1996) and Diamond and Rajan (2000) for an analysis of the optimal capital structure.

$$\max_{\overline{p}_i} \; V = -\overline{K} + \frac{1}{1+W}\int_0^{\hat{p}_i(z)} \Big(S(\overline{p}_i) - p_i(z)\big(S(\overline{p}_i)+\delta+\lambda\big) + \overline{K}\,(1+\delta)\Big)\,dF(p_i(z))$$
$$s.t. \quad \Pr\Big[\overline{K} + \pi(z) \geq k_r\Big] = \alpha_B^* \tag{8}$$

Where $\hat{p}_i(z) \equiv \left\{ \dfrac{S(\overline{p}_i) + \overline{K}(1+\delta)}{S(\overline{p}_i)+\delta+\lambda}, 1 \right\}$ is the bankruptcy rate of default, defined as the one for which the net value of the bank at period 1 is equal to zero, i.e. $p_i(z)$ such that $\pi(z) = 0$. Thus bank's expected net present value can be rewritten as:

$$V = -\overline{K} + \frac{1}{1+W}\left[\int_0^1 \Big(S(\overline{p}_i) - p_i(z)\big(S(\overline{p}_i)+\delta+\lambda\big) + \overline{K}\,(1+\delta)\Big)\,dF(p_i(z))\right] -$$
$$\frac{1}{1+W}\left[\int_{\hat{p}_i(z)}^1 \Big(S(\overline{p}_i) - p_i(z)\big(S(\overline{p}_i)+\delta+\lambda\big) + \overline{K}\,(1+\delta)\Big)\,dF(p_i(z))\right]$$

Or

$$V = -\overline{K} + \frac{1}{1+W}\Big[\Big(S(\overline{p}_i) - \overline{p}_i\,\big(S(\overline{p}_i)+\delta+\lambda\big) + \overline{K}\,(1+\delta)\Big)\Big] -$$
$$\frac{1}{1+W}\left[\int_{\hat{p}_i(z)}^1 \Big(S(\overline{p}_i) - p_i(z)\big(S(\overline{p}_i)+\delta+\lambda\big) + \overline{K}\,(1+\delta)\Big)\,dF(p_i(z))\right] \tag{9}$$

The first term into the brackets represents the net expected return (NER) on the investment, $\Big(S(\overline{p}_i) - \overline{p}_i\,\big(S(\overline{p}_i)+\delta+\lambda\big) + \overline{K}\,(1+\delta)\Big)$, whereas the second term is the option value (OV) of deposit insurance, $-\displaystyle\int_{\hat{p}_i(z)}^1 \Big(S(\overline{p}_i) - p_i(z)\big(S(\overline{p}_i)+\delta+\lambda\big) + \overline{K}\,(1+\delta)\Big)\,dF(p_i(z))$.

3. Optimal asset's risk level choice and regulatory requirements

Given the assumption of constant capital endowment, the objective of the bank is to find the risky asset $\left(\overline{p}_i^*\right)$ that maximizes its value subject to its own target of PD $\left(\alpha_B^*\right)$. Note that the shape of bank's net expected value according to the risky asset $\left(\dfrac{\partial \pi(z)}{\partial \overline{p}_i}\right)$ critically depends on the spreads evolution regarding the class of risk of the project. As we focus on banks' risk taking incentives, we make the parametric assumption on assets such that the bank net expected value is increasing with the class of risk of the investment. We indeed concentrate the analysis on the well known "higher-risk higher-return" (HRHR) assets and we assume:

$$\frac{\partial S(\overline{p}_i)}{\partial \overline{p}_i} > \frac{S(\overline{p}_i)+\delta+\lambda}{1-\overline{p}_i} > 0$$

Moreover, as by assumption, $\dfrac{\partial \overline{K}}{\partial \overline{p}_i} = 0$, we have

$$\frac{\partial V}{\partial \overline{p}_i} \equiv \frac{1}{1+W}\cdot\frac{\partial \pi(z)}{\partial \overline{p}_i} = \frac{1}{1+W}\left[\frac{\partial NER}{\partial \overline{p}_i} + \frac{\partial OV}{\partial \overline{p}_i}\right] > 0, \; \forall \; \overline{p}_i$$

Consequently, the maximization program of banks under the HRHR assets assumption is given by

$$\begin{aligned} &\max_{\overline{p}_i} \quad V = -\overline{K} + \frac{1}{1+W}\int_0^{\hat{p}_i(z)} \pi(z)dF(p_i(z)) \\ &s.t. \quad \Pr\left[\overline{K} + \pi(z) \geq k_r\right] = \alpha_B^* \\ &with \quad \frac{\partial S(\overline{p}_i)}{\partial \overline{p}} > \frac{S(\overline{p}_i) + \delta + \lambda}{1-\overline{p}_i} \end{aligned} \tag{10}$$

3.1. Optimal asset's class of risk

As a solution to (10), the bank will select the investment project which belongs to the last class of risk that satisfies their own target of default probability. Formally the bank retains the last asset's class of risk which satisfies:

$$\Pr\left[\overline{K} + \pi(z) \geq k_r\right] = \alpha_B^*$$

Substituting $\pi(z)$ by its value and rearranging, we obtain equivalently (see appendix 1)

$$\Pr\left[p_i(z) \leq b\right] = \alpha_B^*, \; with \; b \equiv \frac{\overline{K} - k_r + S(\overline{p}_i) + \overline{K}(1+\delta)}{S(\overline{p}_i) + \delta + \lambda} \tag{11}$$

$\dfrac{\overline{K} - k_r + S(\overline{p}_i) + \overline{K}(1+\delta)}{S(\overline{p}_i) + \delta + \lambda}$ represents the critical threshold that the default rate must not exceed in order to satisfy the bank's internal target of default probability. As the default rate $p_i(z)$ is distributed under the cumulative distribution function $F\left(p_i(z)\right)$, we have (see appendix 2):

$$\Pr\left[p_i(z) \leq b\right] = F(b) = \alpha_B^* \text{ and}$$

$$b = \phi\left(\frac{\phi^{-1}(\overline{p}_i) + \sqrt{\rho}\,\phi^{-1}(\alpha_B^*)}{\sqrt{(1-\rho)}}\right) \tag{12}$$

$\phi\left(\dfrac{\phi^{-1}(\overline{p}_i) + \sqrt{\rho}\,\phi^{-1}(\alpha_B^*)}{\sqrt{(1-\rho)}}\right)$ is the distribution of the default rate adjusted to the bank's own target of default probability. The optimal asset's class of risk $\overline{p}_i^*$ retained by the bank will be the one that satisfied the following equation:

$$\phi\left(\frac{\phi^{-1}(\overline{p}_i^*) + \sqrt{\rho}\,\phi^{-1}(\alpha_B^*)}{\sqrt{(1-\rho)}}\right) = \frac{\overline{K} - k_r + S(\overline{p}_i^*) + \overline{K}(1+\delta)}{S(\overline{p}_i^*) + \delta + \lambda} \tag{13}$$

The distribution function of the default rate, adjusted to the bank's own target of default probability, is increasing with both the bank's target and the class of risk the asset belongs to. For convenience we write it $\phi(a)$ with $a = \left(\dfrac{\phi^{-1}(\overline{p}_i) + \sqrt{\rho}\,\phi^{-1}(\alpha_B^*)}{\sqrt{(1-\rho)}}\right)$, which leads to a simpler expression of the condition on optimality of asset's class of risk.

$$\phi\,(a)\left(S(\overline{p}_i^*)+\delta+\lambda\right)-S(\overline{p}_i^*)=\overline{K}-k_r+\overline{K}(1+\delta) \qquad (14)$$

Let's define $f(\overline{p}_i)=\phi\,(a)\left(S(\overline{p}_i)+\delta+\lambda\right)-S(\overline{p}_i)$. $f(\overline{p}_i)$ can be seen as an expression of the expected loss conditional on the default rate adjusted to the bank's internal target of default probability. The shape of this conditional expected loss, which we call hereafter the bank-adjusted expected loss, depends on the variation of spreads regarding the asset's class of risk. Proposition 1 gives the optimal asset choice of the bank whatever the risk sensitivity of regulatory requirements (Basel I or Basel II).

PROPOSITION 1.

a. Assume that $\dfrac{\partial S(\overline{p}_i)}{\partial \overline{p}}<\dfrac{\partial \phi(a)}{\partial \overline{p}}\cdot\dfrac{S(\overline{p}_i)+\delta+\lambda}{1-\overline{p}_i}$ we have $\dfrac{\partial f(\overline{p}_i)}{\partial \overline{p}}>0$. The variation of the spreads rate regarding the class of risk the asset belongs to were such that the bank-adjusted expected loss is increasing with the asset's class of risk.

If, $\overline{K}<\dfrac{(\delta+\lambda)+k_r}{2(1+\delta)}$, there is an interior solution and the optimal class of risk retained by the bank will be such that $\overline{p}_i\in\left]0;\overline{p}_{\max}\right]$.

If $\overline{K}\geq\dfrac{(\delta+\lambda)+k_r}{2(1+\delta)}$, the class of risk retains by the bank would be the riskiest one available $\overline{p}_{\max}$.

b. Assume that $\dfrac{\partial S(\overline{p}_i)}{\partial \overline{p}_i}>\dfrac{\partial \phi(a)}{\partial \overline{p}_i}\cdot\dfrac{S(\overline{p}_i)+\delta+\lambda}{1-\overline{p}_i}$ we have $\dfrac{\partial f(\overline{p}_i)}{\partial \overline{p}_i}<0$. The variation of spreads rate in regards with the class of risk of the asset were such that the bank-adjusted expected loss is decreasing with the class of risk of the asset. The class of risk of the asset retained by the bank would be the riskiest one $\overline{p}_{\max}$. This corner solution would be induced by the HRHR asset assumption ■

Proof of PROPOSITION 1. See appendix 3.

Proposition 1 describes the asset optimal choice of the bank with no special assumption concerning the nature of the capital requirements as k_r can be given by Basel I or Basel II regulatory requirements. However, it is possible to show that regulatory requirements impact the threshold the bank-adjusted expected loss should not exceeds and thus, the optimal class of risk retained at equilibrium by the bank. It means that bank's optimal choice will be impacted by a change in the regulatory environment.

3.2. The regulatory requirements

The first version of the bank capital regulation, the Basel I framework, displayed a risk sensitivity related to the legal category the loan belongs to, such as corporate, sovereign debtors etc., but an invariant risk weighting within a particular legal category, leading to a uniform capital to asset ratio for this category. We translate this invariant regulatory requirements on corporate loans under Basel I capital regulation with: $\overline{k}_r=8\,\%$.

On the contrary, the sensitivity of the revised framework, the Basel II capital regulation, is related not solely to the legal category of the loan but also to its risk level, more precisely to the class of risk to whom the loan is assigned. The regulatory risk weight function and the parameters applied into the computation of regulatory requirements depend on the legal category of the loan (Corporate, Small and Medium Enterprises, Sovereign debtor, etc...). Nevertheless, for a given category, a risk weight function determines the regulatory requirements which are increasing with the risk level of the asset. This risk sensitivity will be reflected by means of: $k_r(\overline{p}_i)$ with $\dfrac{\partial k_r(\overline{p}_i)}{\partial \overline{p}_i} > 0$.

Moreover, for corporate loans, the regulatory weight function, constructed on the ASRF model, gets the reduced form:

$$k_r(\overline{p}_i) = EAD \times LGD \times (p_i(z_{reg}) - \overline{p}_i) \tag{15}$$

With the exposure at default (EAD) equals to 1 in our model; the loss given default (LGD), λ in our framework, a regulatory parameter set to 45% under the foundation IRB approach, and $p_i(z_{reg}) = \phi\left(\dfrac{\phi^{-1}(\overline{p}_i) + \sqrt{\rho}\ \phi^{-1}(0.999)}{\sqrt{(1-\rho)}}\right)$ the default rate adjusted to the target of non-default probability fixed by the regulator.

As a consequence of the different risk sensitivities of the regulation, the optimal class of risk retained by the bank will be different.

In the case of Basel I, $\overline{K} - \overline{k}_r + \overline{K}(1+\delta)$ is constant and the optimal class of risk retained by the bank is the one that fulfilled the following condition

$$\phi\ (a)\left(S(\overline{p}_i) + \delta + \lambda\right) - S(\overline{p}_i) = \overline{K} - \overline{k}_r + \overline{K}(1+\delta) \tag{16}$$

Note that $\overline{k}_r$ should be substituted to k_r in proposition 1 in order to obtain the optimal choice of the bank.

In the case of Basel II, $\overline{K} - k_r(\overline{p}_i) + \overline{K}(1+\delta)$ is decreasing with the asset's class of risk and the optimal class of risk retained by the bank is the one that fulfilled the following condition

$$\phi\ (a)\left(S(\overline{p}_i) + \delta + \lambda\right) - S(\overline{p}_i) = \overline{K} - k_r(\overline{p}_i) + \overline{K}(1+\delta) \tag{17}$$

Note that $k_r\left(\overline{p}_i\right)$ should be substituted to k_r in proposition 1 in order to obtain the optimal choice of the bank.

Depending on the initial banks portfolio risk level, the implementation of the revised framework may modify the regulatory requirement the bank faces. By numerical simulation, we can determine the risk level of the asset which leads to the same regulatory requirements under both regulations and for $\overline{p}_s = 2.269\ \%$ we have $\overline{k}_r = k_r(\overline{p}_s) = 0.08$.

Given that Basel II capital requirements are increasing with the class of risk of the asset, a bank whose portfolio displays a risk level higher than $\overline{p}_s$ will face an increase in its regulatory requirements after the switch of regulation. Conversely, a bank's portfolio composed with asset belonging to a class of risk inferior to $\overline{p}_s$ will benefit from a reduction in the regulatory requirements. It means that the change in the value of the capital requirements incites bank to change its behaviour and thus impact the optimal level of risk of its asset. Actually, we show that the impact of the regulatory requirements on the choice of asset's class of risk lies on the initial capital level the bank gets to comply with its own target of default probability.

3.3. The impact of the regulatory risk sensitivity on the asset risk level choice

We define $\overline{K}(\overline{p}_s)$ as the initial capital endowment that leads the bank to select the class of risk $\overline{p}_s$ under Basel I capital requirements that is:

$$\phi\ (a_{\overline{p}_s})\left(S(\overline{p}_s) + \delta + \lambda\right) - S(\overline{p}_s) = \overline{K}(\overline{p}_s) - \overline{k}_r + \overline{K}(\overline{p}_s)(1+\delta)$$

It is equal to:

$$\overline{K}(\overline{p}_s) = \frac{\phi\,(a_{\overline{p}_s})\left(S(\overline{p}_s)+\delta+\lambda\right)-S(\overline{p}_s)+\overline{k}_r}{(2+\delta)} \tag{18}$$

In the same line, we determine $\overline{K}(\overline{p}_{\max})$ as the capital endowment allowing the bank to opt for the asset assigned to the riskiest class $\overline{p}_{\max}$. Under the Basel II capital regulation, $\overline{K}(\overline{p}_{\max})$ must satisfy

$\phi\,(a_{\overline{p}_{\max}})\left(S(\overline{p}_{\max})+\delta+\lambda\right)-S(\overline{p}_{\max}) \leq \overline{K}(\overline{p}_{\max})-k_r(\overline{p}_{\max})+\overline{K}(\overline{p}_{\max})\,(1+\delta)$ which lead to

$$\overline{K}(\overline{p}_{\max}) = \frac{\phi\,(a_{\overline{p}_{\max}})\left(S(\overline{p}_{\max})+\delta+\lambda\right)-S(\overline{p}_{\max})+k_r(\overline{p}_{\max})}{(2+\delta)} \tag{19}$$

PROPOSITION 2.

1. If $\overline{K} < \overline{K}(\overline{p}_s)$ the bank will increase the level of risk of its equilibrium asset after a shift from Basel I to Basel II.
2. If $\overline{K}(\overline{p}_s) < \overline{K} < \overline{K}(\overline{p}_{\max})$ the bank will decrease the level of risk of its equilibrium asset after a shift from Basel I to Basel II.
3. If $\overline{K} > \overline{K}(\overline{p}_{\max})$ the bank keeps unchanged its optimal asset class of risk which is already the maximum one $\overline{p}_{\max}$ ■

The proof of proposition 2 is the following.

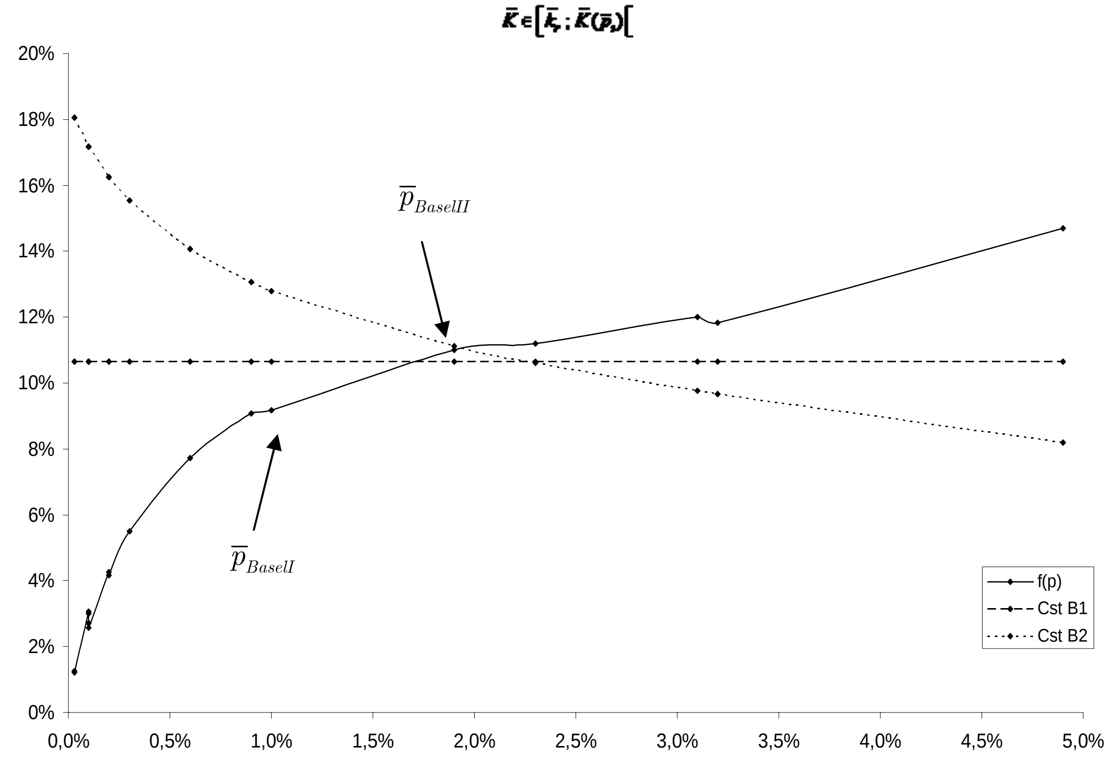
Case 1. Bank initial capital endowment is inferior to $\overline{K}(\overline{p}_s)$ and the optimal asset's class of risk retained under Basel I $\overline{p}_i^*$ is inferior to $\overline{p}_s$. Given that the Basel II capital requirements are increasing with the class of risk of the asset, this leads that the implementation of the revised framework decreases the regulatory requirements the bank faces. What can be the reaction of a value maximizing bank which faces a freeing of capital? The class of risk $\overline{p}_i^* < \overline{p}_s$ that was optimally selected under Basel I is thus no longer the equilibrium one as $\phi\,(a)\left(S(\overline{p}_i^*)+\delta+\lambda\right)-S(\overline{p}_i^*) < \overline{K}-k_r\left(\overline{p}_i^*\right)+\overline{K}(1+\delta)$. Consequently, the bank reallocates its asset in order to comply with its objectives: maximising its value while respecting its internal target of risk. Under our assumption of HRHR, these objectives lead to a selection of a riskier asset $\overline{p}_j^* > \overline{p}_i^*$ under Basel II such that $\phi\,(a)\left(S(\overline{p}_j^*)+\delta+\lambda\right)-S(\overline{p}_j^*) = \overline{K}-k_r\left(\overline{p}_j^*\right)+\overline{K}(1+\delta)$. Hence, this lightening in the regulatory charges induces the bank to choose for an asset assigned to a riskier class of risk in order to increase its value (cf. graph 1 for illustrative purpose).

Case 2. The initial capital endowment is comprised between $\overline{K}(\overline{p}_s)$ and $\overline{K}(\overline{p}_{Max})$. The Basel I optimal choice of asset's class of risk $\overline{p}_i^*$ is superior to $\overline{p}_s$ inducing an increase in the regulatory charges after the switch in regulation. However, the level of capital endowment is not high enough to both absorb the increase in capital requirements and still comply with the internal target of default probability. In the same line of reasoning than before, this strengthening of the regulatory constraint leads the bank to opt for a less risky asset in order to respect its own target of default probability (cf. graph 2).

Case 3. Finally, if the capital endowment is superior to $\overline{K}(\overline{p}_{Max})$, the Basel I optimal asset's class of risk retained by the bank $\overline{p}_i^*$ is riskier than $\overline{p}_s$ (actually it is the maximum level of risk $\overline{p}_{\max}$, see proposition 1). Although this leads to an increase in the regulatory requirements after the implementation of Basel II, the level of capital of the bank is high enough to absorb this increase with respect to its own target of default probability. The bank thus keeps its portfolio unchanged and preserves its value.
We can depict these mechanisms for specific values of the spread extracted from Resti and Sironi (2007). We use a riskless interest rate of 5% $(\delta = 0.05)$, a LGD of 45 % as in Basel II $(\lambda = 0.45)$ and $\alpha_B^* = 99.99\%$.[8] As the classes of risk available for the bank are not continuous, its optimal choice will be the last one that satisfies its own target of default probability, that is:

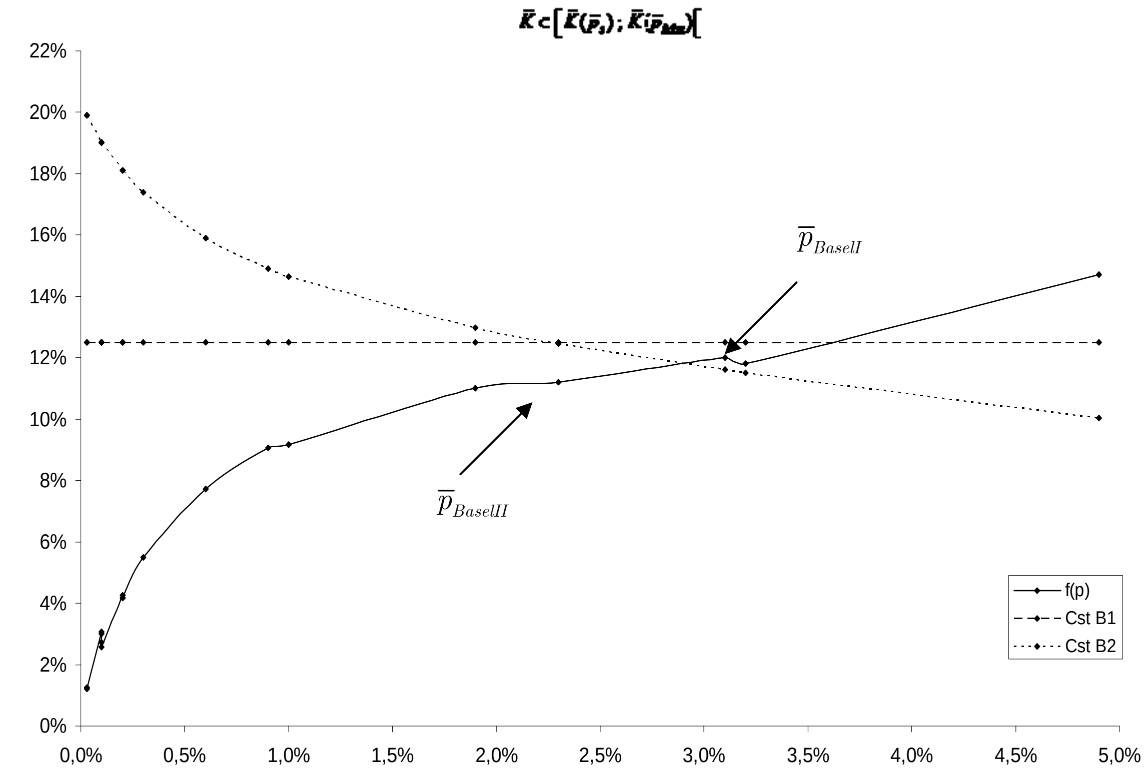
$$\phi\left(\frac{\phi^{-1}(\overline{p}_i) + \sqrt{\rho}\ \phi^{-1}(\alpha_B *)}{\sqrt{(1-\rho)}}\right) \quad \leq \quad \frac{\overline{K} - k_r + S(\overline{p}_i) + \overline{K}(1+\delta)}{S(\overline{p}_i) + \delta + \lambda}$$

Case 1.

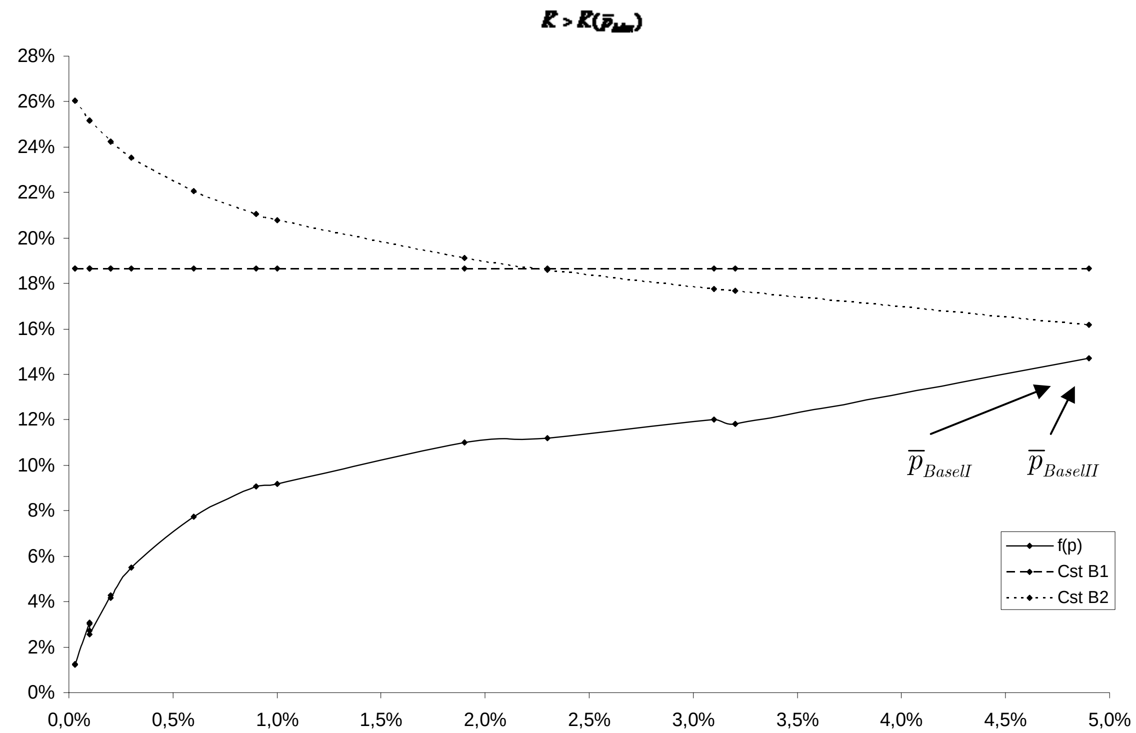


8. Equivalent to a AAA rating.

Case 2.


22%
20%
18%
16%
14%
12%
10%
8%
6%
4%
2%
0%
0,0%
0,5%
1,0%
1,5%
2,0%
2,5%
3,0%
3,5%
4,0%
4,5%
5,0%
$\overline{p}_{BaselI}$
$\overline{p}_{BaselII}$
f(p)
Cst B1
Cst B2

Case 3.



4. Procyclicality

Many studies concentrate their attention to cyclical effects of regulatory requirements, analysing the lending behaviour of banks along the business cycle. They conclude that capital requirements impact banks lending behaviour amplifying the cyclicality of the economy, phenomenon known as procyclicality. Moreover, the risk sensitivity of regulatory requirements accentuates the procyclicality: during economic downturns, risk estimations of borrowers are severer than under economic upturns. This induces higher capital requirements while economic conditions decline and leads banks to cut their credit supply, worsening thus the recession. The reverse happens during economic upturns, where regulatory requirements decrease while economic conditions improve, allowing thus banks to boost their loans supply and exacerbating the economic boom.

Such analysis, detailed in the introduction, are generally done in the literature by studying the asset side of banks portfolio with segregation between loans and bonds, adding to the assumption of a positive amount of regulatory requirements assigned to loans and none for bonds; distinction we do not operate in our model. Assuming indeed an exposure at default equates to one allows us concentrating our analysis on the impact of risk sensitivity capital requirements on the choice of asset's risk level. The question we address is thus the following: what would be, along the business cycle, the impact of the regulatory risk sensitivity on the asset's risk level retained by banks?

To study this point, we add the assumption that the borrower's default probability assessed by the bank depends on the relative position of the economy within the business cycle: during economic upturns, estimations of borrowers' default probabilities are more favourable than during economic downturns. We translate this relation between the business cycle c, and the firms' risk level $\overline{p}_i$ by means of $\overline{p}_i(c)$, assuming that $\frac{\partial \overline{p}_i(c)}{\partial c} < 0$.

As we already stress, within a particular regulatory category, Basel II capital requirements are risk sensitive whereas Basel I regulatory charges are risk invariant. Consequently the cyclicality of the economy affects the Basel II regulatory requirements while leaving unchanged the Basel I regulatory requirements:

In Basel I, $\overline{k}_r$ is independent of $\overline{p}_i$ and $\frac{\partial \overline{k}_r}{\partial \overline{p}_i(c)} = 0$ which means $\frac{\partial \overline{k}_r}{\partial c} = \frac{\partial \overline{k}_r}{\partial \overline{p}_i(c)} \cdot \frac{\partial \overline{p}_i(c)}{\partial c} = 0$.

In Basel II, we have $k_r(\overline{p}_i(c))$ and $\frac{\partial k_r(\overline{p}_i(c))}{\partial \overline{p}_i(c)} > 0$ which means

$\frac{\partial k_r(\overline{p}_i(c))}{\partial c} = \frac{\partial k_r(\overline{p}_i(c))}{\partial \overline{p}_i(c)} \cdot \frac{\partial \overline{p}_i(c)}{\partial c} < 0$.

These variations in regulatory requirements impact the threshold the optimal asset's class of risk must not exceed (referee to equations (16) and (17)). Under Basel I, the threshold remains constant, equals to $\overline{K} - \overline{k}_r + \overline{K}(1+\delta)$ whereas under Basel II, this constraint becomes positively related to the business cycle, softening as the economic conditions improve ($c+$), strengthening while they deteriorate ($c-$) and

$\overline{K} - k_r\left(\overline{p}_i(c-)\right) + \overline{K}(1+\delta) \leq \overline{K} - k_r\left(\overline{p}_i(c+)\right) + \overline{K}(1+\delta)$

On the other side, credit spreads and default rates, i.e. default probabilities conditional on the realization of the systematic risk factor, also depend on the business cycle. As we make the HRHR asset assumption, credit spreads are negatively linked with the business cycle: while economic conditions improve, credit spreads decrease, reflecting an easier access to liquidity and higher expectations on investment returns; reverse happening while economic conditions deteriorate. This result is formally express by the following equation:

$$\text{as } \frac{\partial S\left(\overline{p}_i(c)\right)}{\partial \overline{p}_i(c)} > 0, \forall c \text{, we have } \frac{\partial S\left(\overline{p}_i(c)\right)}{\partial c} = \frac{\partial S\left(\overline{p}_i(c)\right)}{\partial \overline{p}_i(c)} \cdot \frac{\partial \overline{p}_i(c)}{\partial c} < 0 \qquad (20)$$

In the same line of thinking, the default rate of a project is also negatively related to the business cycle, decreasing as the economic conditions improve. Reminding that the default rate adjusted to the bank's own target of default probability is distributed according to

$\phi(a) = \phi\left(\frac{\phi^{-1}\left(\overline{p}_i(c)\right) + \sqrt{\rho}\ \phi^{-1}\left(\alpha_B^*\right)}{\sqrt{(1-\rho)}}\right)$ with $\frac{\partial \phi\left(a\left(\overline{p}_i(c)\right)\right)}{\partial \overline{p}_i(c)} > 0$,[9] we have

$$\frac{\partial \phi\left(a\left(\overline{p}_i(c)\right)\right)}{\partial c} = \frac{\partial \phi\left(a\left(\overline{p}_i(c)\right)\right)}{\partial \overline{p}_i(c)} \cdot \frac{\partial \overline{p}_i(c)}{\partial c} < 0 \qquad (21)$$

Then recalling that under Basel I, the optimal asset's class of risk is such that:

$$\phi\left(a\left(\overline{p}_i(c)\right)\right)\left(S\left(\overline{p}_i(c)\right) + \delta + \lambda\right) - S(\overline{p}_i) = \overline{K} - \overline{k}_r + \overline{K}(1+\delta)$$

We can synthesize the effects of the economic cycle on the choice of optimal asset's class of risk by the bank. In order for the initial equilibrium choice of the bank under Basel I to be invariant with the cycle, we must have following condition

9. As ϕ denotes the cumulative distribution function of a standard normal random variable, it is by definition increasing with the risk level of the asset.

$$\frac{\partial\phi\left(a\left(\overline{p}_i(c)\right)\right)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(S\left(\overline{p}_i^*(c)\right)+\delta+\lambda\right)-$$
$$\frac{\partial S\left(\overline{p}_i(c)\right)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(1-\phi\left(a\left(\overline{p}_i^*(c)\right)\right)\right)=0$$

Or equivalently

$$\frac{\partial\phi\left(a(.)\right)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(S\left(\overline{p}_i^*(c)\right)+\delta+\lambda\right)=$$
$$\frac{\partial S(.)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(1-\phi\left(a\left(\overline{p}_i^*(c)\right)\right)\right) \qquad (22)$$

Equation (22) means that, in order for the bank to keep unchanged its initial choice of asset, the variation in the expected return in case of investment failure, $\frac{\partial\phi\left(a\left(\overline{p}_i(c)\right)\right)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(S\left(\overline{p}_i^*(c)\right)+\delta+\lambda\right)$, must be overbalanced by the variation in the expected return in case of project success $\frac{\partial S\left(\overline{p}_i(c)\right)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(1-\phi\left(a\left(\overline{p}_i^*(c)\right)\right)\right)$.

When economic conditions improve, both of them are negative. Consequently the decrease in the expected return in case of success, due to a decline in credit spreads, should be of lesser magnitude than the drop of the expected return in case of failure, for the initial asset's class of risk to remain optimal for the bank.

Conversely, both elements of (22) are positive when economic conditions worsen. The rise in expected returns in case of success, due to an increase in credit spreads, must compensate the higher expected returns in case of investment failure, for the asset's class of risk to remain optimal for bank.

With the same line of thinking, the optimal asset's class of risk is reached in Basel II when:

$$\phi\left(a\left(\overline{p}_i(c)\right)\right)\left(S\left(\overline{p}_i(c)\right)+\delta+\lambda\right)-S(\overline{p}_i(c))=\overline{K}-k_r\left(\overline{p}_i(c)\right)+\overline{K}(1+\delta)$$

Consequently, the initial equilibrium choice of the bank under Basel II is invariant with the cycle if,

$$\frac{\partial\phi\left(a\left(\overline{p}_i(c)\right)\right)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(S\left(\overline{p}_i^*(c)\right)+\delta+\lambda\right)-$$
$$\frac{\partial S\left(\overline{p}_i(c)\right)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(1-\phi\left(a\left(\overline{p}_i^*(c)\right)\right)\right)=-\frac{\partial k_r\left(\overline{p}_i(c)\right)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)$$

Or equivalently:

$$\frac{\partial\phi\left(a(.)\right)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(S\left(\overline{p}_i^*(c)\right)+\delta+\lambda\right)= \qquad (23)$$
$$\frac{\partial S(.)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(1-\phi\left(a\left(\overline{p}_i^*(c)\right)\right)\right)-\frac{\partial k_r(.)}{\partial\overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial\overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)$$

Note that condition (23) is similar to the one obtain under Basel I capital regulation, except the added of the term $-\frac{\partial k_r(\overline{p}_i(c))}{\partial \overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)$. This term adjusts the expected return in case of success with a "regulatory bonus" when the business cycle improve and / or when the asset's class of risk retained is riskless, and conversely, with a "regulatory penalty" when a risky asset is selected by the bank and / or when the economic conditions deteriorate.
Actually, during economic downturns, credit spreads, default rates and regulatory requirements increase. As a consequence, in order for the initial class of risk to remain optimal for the bank, the trade off depicted by (23) requires that the increase in credit spreads, which raises the expected returns in case of success, must overbalances both the raise in the expected returns in case of project failure and the "regulatory penalty".
On the contrary, when economic conditions improve, spreads rate, default rate and regulatory capital charges decrease. In order for the initial class of risk to remain optimal for the bank, the fall in expected returns in case of investment success, partially or fully compensated by the "regulatory bonus" must not be higher than the decline in expected returns in case of project failure. Variations in Basel II regulatory requirements due to economic cycle interact with the trade off initially depicted under Basel I, strengthening it when economic conditions deteriorate, softening it while business circumstances improve. Proposition 3 summarizes the cyclical impact of risk sensitivity requirements on the optimal asset class of risk, for a constant capital endowment:

PROPOSITION 3.
Assume a change in the business cycle (∂c) and a continuum of classes of risk, $\overline{p}_i \in \left]0;\overline{p}_{\max}\right]$ with $\overline{p}_{\max} < 1$.

1. If $\frac{\partial \phi(a(.))}{\partial \overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(S\left(\overline{p}_i^*(c)\right)+\delta+\lambda\right) <$

$$\frac{\partial S(.)}{\partial \overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(1-\phi\left(a\left(\overline{p}_i^*(c)\right)\right)\right)-\frac{\partial k_r(.)}{\partial \overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)$$

The bank will opt for a riskier asset $\overline{p}_j^* > \overline{p}_i^*$ with $\overline{p}_j^*$ such that

$$\frac{\partial \phi(a(.))}{\partial \overline{p}_i(c)}\left(\overline{p}_j^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_j^*(c)\right)\cdot\left(S\left(\overline{p}_j^*(c)\right)+\delta+\lambda\right) =$$

$$\frac{\partial S(.)}{\partial \overline{p}_i(c)}\left(\overline{p}_j^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_j^*(c)\right)\cdot\left(1-\phi\left(a\left(\overline{p}_j^*(c)\right)\right)\right)-\frac{\partial k_r(.)}{\partial \overline{p}_i(c)}\left(\overline{p}_j^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_j^*(c)\right)$$

2. If $\frac{\partial \phi(a(.))}{\partial \overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(S\left(\overline{p}_i^*(c)\right)+\delta+\lambda\right) >$

$$\frac{\partial S(.)}{\partial \overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)\cdot\left(1-\phi\left(a\left(\overline{p}_i^*(c)\right)\right)\right)-\frac{\partial k_r(.)}{\partial \overline{p}_i(c)}\left(\overline{p}_i^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_i^*(c)\right)$$

The bank will opt for a less risky asset $\overline{p}_k^* < \overline{p}_i^*$ with $\overline{p}_k^*$ such that

$$\frac{\partial \phi(a(.))}{\partial \overline{p}_i(c)}\left(\overline{p}_k^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_k^*(c)\right)\cdot\left(S\left(\overline{p}_k^*(c)\right)+\delta+\lambda\right)=$$
$$\frac{\partial S(.)}{\partial \overline{p}_i(c)}\left(\overline{p}_k^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_k^*(c)\right)\cdot\left(1-\phi\left(a\left(\overline{p}_k^*(c)\right)\right)\right)-\frac{\partial k_r(.)}{\partial \overline{p}_i(c)}\left(\overline{p}_k^*(c)\right)\cdot\frac{\partial \overline{p}_i(c)}{\partial c}\left(\overline{p}_k^*(c)\right)$$

Proposition 3 is easily understandable as it answers to the following question: what's happens when equilibrium condition (23) is no longer fulfilled?
Actually, the bank must be incited to change its behaviour and choose a riskier or lower risk level as optimal. Two situations are possible. In the case of an economic expansion, the lightening in regulatory capital charges creates a "regulatory bonus" for the bank. The softening of regulatory requirements impacts all classes of risk. Furthermore, the freeing of regulatory capital for the bank onto the particular class of risk initially retained, associated with the lightening of regulatory requirements for all classes of risk, might incite the bank to opt for a new class of risk, exploiting the gain of regulatory capital to allocate across a riskier class of asset to increase its value (case 1 of proposition 3). Within the same line of thinking, additional regulatory requirements due to worsen economic conditions impact all asset classes of risk. This "regulatory penalty" would be of such amount that it overbalances the increase in expected returns and leads the bank to select for a new class of risk, reducing the risk level retained to comply with its own target of default probability (case 2 of proposition 3).
However, this process of asset reallocation is always possible in our setting only because we assume that the bank can choose its optimal level of risk in a continuous range of assets $\overline{p}_i \in \left]0;\overline{p}_{\max}\right]$. Actually, in a more realistic setting, the granularity of the risk bucket available to the bank is not infinite, and borrowers sharing different risk profile are to be clustered into the same risk bucket. The risk bucket PD[10] represents the regulatory input used to compute capital requirements, and leads to thresholds effects. In that case, for instance, the switch towards a riskier asset implies a discontinuous jump from a risk bucket to a riskier one. Depending on the granularity of the bank's risk bucket, this jump towards a riskier bucket may raise regulatory requirements of such an amount that bank cannot comply with the constraint for that risk level. In this case, where the next risk bucket leads to an infringement of the constraint, the bank keeps its current asset even if the risk level is sub-optimal (i.e. not binding with the constraint). The same reasoning applies in case of economic recession. If the regulatory "penalty", related to the current risk bucket the asset belongs to, is of such an amount that this risk level is no more optimal, there is an incentive for the bank to move towards a lower risk bucket. However, because of risk bucket granularity, the bank must be obliged to opt for the first risk bucket that complies with the constraint even if this leads to a selection of a risk level that is suboptimal (i.e. not binding with the constraint).
The traditional literature concludes to the pro-cyclical impact of capital requirements in terms of volume of lending, as on the exacerbation of this phenomenon by the risk sensitivity embodied into the revised regulatory ratio. Our analysis stresses the pro-cyclical impact in terms of risk level induced by the risk sensitivity of the revised ratio. In addition to the trade off between the variations in expected returns in case of failure or success of the project existing under Basel I, is added either a "regulatory bonus" which can be exploited by the bank or a "regulatory penalty" that restricts the bank. As economic conditions improve, the gain in regulatory capital can be used by the bank to increase its value, retaining a riskier class of asset and exacerbating the economic boom. If such a scenario happens, the rise into the risk level of bank portfolio leads to a weakening of the bank which is similar to an increase in financial fragility. In case of deterioration

10. Defined as the average or the mean of borrowers' PD; we discuss latter whether the bank internal rating system is either PIT or TTC impacts our result.

in business conditions, the additional regulatory charges can constrain the bank to reduce its risk level, amplifying the credit crunch and the recession. This lowering into the risk level of the bank portfolio represents a "fly to quality" phenomenon.

5. Discussion

The analysis conducted in this model investigates the impact the risk sensitive regulatory capital to asset ratio may have on banks' risk taking behaviours. Two aspects are analysed: we first concentrate on the potential effects induced by the implementation of a risk sensitive ratio while the flat rate requirements prevailed. We then study the cyclical impact that could affect bank risk taking behaviour according to the risk sensitivity of the regulatory requirements. To the initial trade-off between marginal variations in expected returns in case of investment failure or success, which may drive the risk taking behaviour of the bank along the business cycle, the risk sensitivity of the regulatory requirements adds either a "regulatory bonus" or a "regulatory penalty". Depending on the magnitude of the cyclical variations into regulatory requirements, the "bonus" may be exploited by the bank to increase its value toward the selection of a riskier asset when economic conditions improve, whereas when recession occurs, the "penalty" may restrict the bank to opt for a less risky asset. Whether the optimal asset risk level swings from a class of risk to another through the cycle, the financial fragility may increase during economic upturns.

The sensitivity of regulatory parameters, i.e. the default probability related to borrowers' class of risk for foundation IRB approach, added to the exposure at default (EAD) and the loss given default (LGD) for advanced IRB approach, depends on banks internal system. Mainly two rating systems exist: PIT and TTC, with the former being more sensitive to cyclical variations than the latter. Higher sensitivity seems favour the emergence of a new class of risk as optimal: higher sensitivity of such regulatory parameters induced more sensitive estimations to business cycle fluctuations that encourage the swing among asset's classes of risk. Banks using a PIT rating system may be more inclined to adjust the risk level of their loans portfolio along the business cycle than TTC rating system operating banks, converging to the established result of higher procyclicality under PIT approach, but in terms of asset risk level instead of loans volume.

The shareholders' dominated ownership structure and the distribution of asset earnings of our setting exacerbate risk taking incentives. However the analysis made by Jeitschko and Dong Jeung (2005) gives us two interesting results.

First, they conclude that a risk sensitive regulation leads to the convergence into the respective objective of the main bank liability owners. Actually shareholders seek to maximize the value of their capital, whereas the regulator, on behalf of depositors, aims to avoid bankruptcy. The risk sensitive capital regulation, involving higher requirements for higher risk level, brings closer shareholders and regulator respective objectives. An increase in capital requirements curbs shareholders risk appetite. However, increased capital requirements minimize the bank probability of failure, involving for some regulatory forbearance. Consequently, under the risk sensitive regulation, the optimal risk level chosen by shareholders and regulator might be closer: compare to the flat rate ratio, shareholders opt for a lower risk level whereas regulator might concede an increase in its optimal risk level.

Second, they highlight that earnings distribution displaying a constant or a decreasing mean of returns for an increasing volatility as the risk level (α) of the asset growths, i.e. the traditional mean ordering distribution with $\mu'(\alpha) \leq 0$; $\sigma'(\alpha) \geq 0$, leads the shareholders to favour those higher returns / lower risk assets under a risk sensitive regulation. Those risk profiles imply higher requirements for an unchanged or a decreasing mean of returns when the risk level increases. Under a risk sensitive regulation, shareholders optimal risk level for such earning distribution might incur a significant drop compare to risk insensitive capital ratio. To allow for the emergence a risk taking behaviour, the HRHR asset risk profiles must be retained.

Although our setting with shareholders-owners banks and HRHR asset risk profile exacerbates risk taking behaviour, the convergence of the optimal risk level selected by each owner of bank liability under risk sensitive requirements supports the view that the ownership structure of bank capital may be less decisive than under a flat rate ratio.

Our result concerning the cyclical impacts of risk sensitive regulatory requirements derives from a comparative static analysis under a constant capital endowment. This constant capital assumption contrasts with other settings that assume a free adjustable leverage for the bank. Moreover our setting gives an interesting sight with regard to the Basel III novelties: the leverage ratio and the capital buffer conservative and countercyclical policies.[11]

The leverage ratio acts as a backstop ratio the bank capital shall not fall below (fixed at 3%). As it aims to give a complementary measure of the risk of the bank, the Basel Committee defines it as a risk insensitive measure: the capital (numerator) as exposure (denominator) are denominated in amount without being related to the risk weighted asset computed under the first pillar. Our setting examines how risk sensitive requirements impact along the business cycle the risk taking behaviour of a bank at constant capital endowment: the leverage ratio is constant in our setting and above the leverage ratio prescribed by the banking regulation.

Instituting such a leverage ratio aims to prevent from excessive leverage and tries to deal with systemic risk: it aims that capital would not fall below a minimum to maintain a shock absorbing capability from the bank. However it does not interact with the cyclical impacts induced by the risk sensitive requirements as presented by our analysis. The cyclical variations into regulatory requirements affect bank's risk taking behaviour, fostering its risk appetite as regulatory requirements soften, restricting it as regulatory requirements strengthen, without modifying the leverage ratio in our setting (constant capital level and exposure set at one by assumption). The leverage ratio seems to be a complementary measure of the risk of the bank in a risk-wide managing perspective but does not prevent from a risk taking behaviour induced by the business cycle variations into the regulatory requirements.

The conservative and the countercyclical buffers are both defined as a 2.5% additional requirement of the risk weighted asset. Both buffers pursue the objective that banks operate with capital level well above the minimum required by the first pillar such that the bank can absorb financial shocks. More precisely both policies require the bank to build up capital buffer during economic upturns that can be draw down as losses occur during business downturns. The conservative buffer applies at each expansion period whereas the countercyclical buffer occurs only under the supervisory decision, when credit growth is perceived as excessive by the supervisory authorities. They define a corridor of additional capital requirements the bank must comply with, otherwise a penalty, consisting in restricting the earning to be distributed as dividend, applies. They complete the first pillar and the leverage ratio in their objective of avoiding a spill over of financial shocks to the real economy by maintaining a level of capital high enough to absorb shock.

Our analysis stresses the level of capital that excesses regulatory requirements plays a crucial role to allow the bank exploiting the regulatory "bonus" by means of a risk taking behaviour that increases its value as business conditions improve. Instituting a conservative buffer the bank must build up and maintain as good business conditions prevail, counteracts the risk taking opportunity offered by the regulatory "bonus". Only very highly capitalized banks could exploit the regulatory "bonus" under a conservative buffer policy. Less well capitalized banks that face this additional requirement may transfer the "bonus" obtained from an alleviate in the regulatory ratio to the

11. Basel Committee on Banking Supervision (2010)

conservative buffer requirement, otherwise they would have to raise capital to increase their optimal asset risk level and comply with these two requirements.
Although our setting cannot precisely examine what the countercyclical buffer implies on our result, we think that it reinforces the risk restrictive impact of the conservative buffer. Whether a very highly capitalized bank could exploit the regulatory "bonus" and still complies with both regulatory ratio and conservative buffer requirements, the occurrence of this further countercyclical buffer may restrict the bank risk taking behaviour, inducing the bank to raise capital to comply with those three requirements that could equals 13% of the risk weighted asset.[12]
The regulatory "penalty" occurring as business conditions worsen leads the bank to lower its optimal risk level in our setting. The conservative and the countercyclical buffers prevail only during expansion phase, the bank being no longer required to maintain these additional requirements during economic downturns. Moreover the conservative and the countercyclical buffers aim to be drawn down as bank incurs losses. These additional regulatory capital requirements, inherited from the expansion period, may help the bank to comply with the first pillar as its own target of default without modifying its asset choice. Those buffers may help avoiding or diminishing the "fly to quality" phenomenon that could occurs under the recession.
The Committee defines the buffers to a risk-wide management perspective, aiming that the capital level of individual banks would be high enough to prevent from a widening of shocks to the financial as the real sectors. Therefore the design of these policies, applying as economic conditions get better, helps counteracting the risk taking opportunities induced by the risk sensitive requirements that vary with the business cycle. Only very highly capitalized bank could insulate from the restrictive impacts of the buffer policies and exploit the regulatory "bonus", if we reason under a constant capital level.
Admittedly, we exogenously set the capital endowment in our framework and we assume it to be constant. For a short horizon of analysis as for comparative statics, these assumptions seem reasonable. Nevertheless, to inquire the risk taking behaviour along the cycle, it would be interesting to investigate the optimal leverage choice of the bank. As the optimal leverage impacts the optimal risk level of loans portfolio, and reciprocally, the optimal risk level of loans portfolio influences the optimal leverage of bank liability, the dynamic analysis of risk taking behaviour along the business cycle would ideally require a framework that endogenously determines both the optimal leverage ratio as the portfolio risk level. The bank internal target of default probability might also interact with those elements as it depends on both the optimal portfolio risk level and the capital leverage.
The analysis conducted by Milne and Whalley (2001) represents a first spet toward this challenge, as they endogenously determine the capital level but they made comparative statics and do not retain an internal target of default probability. Moreover, they assume an asset side of bank portfolio that is fixed and not explicitly modelled. Their result stresses the role the franchise value may play on risk taking behaviour, mitigating the risk taking incentives. However the franchise value, which reflects asset future earnings, directly depends on the risk level of asset selected by the bank in the current period. Although the non linear relation between franchise value and bank desired level of capital they establish in their setting seems taking into consideration that higher returns bring higher franchise value and provides an earning protection against regulatory infringements, they do not integrate this intuition in their risk taking behaviour comparative statics. The intuition that the existence of a franchise value may induce the bank to act prudently to preserve it, as Milne and Whalley (2001) highlight, competes with the idea that an increase in the risk level today fosters future asset earnings and increases bank franchise value, as Blum (1998) and Hellman and al (2000) stress.

12. Respectively 8% for the first pillar and 2.5% for each buffer.

The HRHR risk profile of our framework implies, in the scenario of a rise in asset risk level, an increase in both the current and the next period bank value with still a compliance on regulatory requirements and bank own target of default probability. The latter effect of franchise value described by Blum (1999) and Hellman and al (2000) seems prevail on the former in our setting. Actually all depends on the cyclical pattern the bank expects for the next period economic conditions, as the potential adjustment on leverage ratio the bank can do.
The dynamic analysis conducted by Estrella (2004) brings some answers to this issue. His study focuses on the cyclical effects risk sensitive capital requirements, like a value-at-risk capital ratio, may have on the optimal capital target selected along the business cycle by a bank that optimizes over costs associated with failure, holding capital, and flows of external capital. Although he does not examine the asset risk level optimally selected, he incorporates into his infinite-horizon optimization program an optimal probability of failure that is endogenously determined.
He starts from the assumption that the business cycle is a predictable pattern that unfolds over time which should be incorporated into the decision of capital target by a forward looking bank. Without detailing his study, we present the intuition that conducts the bank decision program and his main results. As costs prevail on both directions into capital level adjustment (when bank raises or sheds capital), it appears optimal for the bank to lose capital at the peak of the recession, for absorbing borrowers default, instead of increasing capital. In the same thinking, it reveals optimal for the bank to build up capital while the expansion prevails. The lag between the risk sensitivity of regulatory requirements, which are in adequacy with the cyclical pattern of asset returns, and bank's capital level target stands on the adjustment costs related to external capital flows.
Estrella (2004) stresses the divergence between the regulatory requirements level and bank optimal target: while economic conditions deteriorate and recession occurs, risk sensitive requirements increase whereas optimal bank capital level decreases, absorbing borrowers default. At the peak of economic downturn, which announces the reversal into economic cyclicality toward the expansion phase, although capital requirements remain at important level, they begin to decrease to reflect the improvement of the business environment. Bank capital target however continues to decrease, the cost minimizing objective pushes the bank to absorb losses on capital rather raise capital since it is costly. The credit crunch may occur during this second sequence of the economic downturns as risk capital requirements may exceed bank optimal target level, encouraging a shift toward low risk asset that soften regulatory requirements.
When economic expansion arises, regulatory requirements decrease to reach their lowest level at the peak of the economic upturn. During the emergence of the good period, the bank optimal capital target increases: the adjustment costs make desirable to raise new capital while business conditions improve, as asset earnings increase and compensate the adjustment cost of capital level. At the highest level of expansion phase, the cyclical pattern predicts a reversal into economic environment toward a recession period, which leads the risk sensitive requirements to increase, but it remains optimal for the bank to build up capital during this ending of economic upturn. As a consequence the optimal capital target is well above the regulatory requirements at the peak of the expansion. Estrella (2004) highlights the moral hazard problem that may be exacerbated when expansion prevails under risk sensitive capital requirements.
Our setting assumes a constant capital endowment higher than regulatory requirements. Under such a restrictive assumption, we depict how the risk sensitive ratio may induce risk taking behaviour: as economic upturn occurs and frees regulatory capital, the bank can exploit the "regulatory bonus" to increase its value toward the selection of a riskier asset. The results of Estrella (2004) reinforce the idea that moral hazard problem may exists under risk sensitive requirements for a bank that selects its optimal level of capital along the business cycle.

Adjustment costs create a lag between the optimal capital level target and risk sensitive requirements. This involves diverging cyclical evolutions of the bank capital target and risk sensitive requirements: capital target increases as expansion unfolds and decreases while recession prevails, i.e. from the beginning to the end of the expansion / recession phase, whereas risk sensitive requirements decrease as economic conditions gets better and increase as business condition worsen; i.e. from an expansion (recession) peak to a recession (expansion) peak. These diverging cyclical behaviours create a gap between capital level and requirements that is highest at the peak of expansion phase, when risk assessment parameters are lowest. Assuming the bank endogenously determines its leverage along the business cycle seems in favour of our intuition, risk taking behaviour may occur when expansion prevails. As the bank optimal leverage is well above regulatory requirements at the peak of expansion phase, the bank can decide to exploit its excess of capital to foster its value toward the selection of a riskier asset. The resulting weakening of bank safety may accentuate the loss the bank faces when recession happens, particularly in a dynamic setting where it is optimal for the bank to absorb losses on capital instead of raising it. Our intuition that the "regulatory penalty" brought by risk sensitive requirements further restricts bank risk behaviour and pushes it to opt for an even lower asset's risk level, leads to a "fly to quality" phenomenon that exacerbates the credit crunch.

The analysis of Estrella (2004) reinforces the crucial role of the buffer policies. However these buffers requirements that apply while expansion prevails can be integrated into the bank's decision program as the level of requirements the bank must comply with in expansion. They do not alter the cyclical behaviour of the bank's optimal leverage target depicted into the analysis conducted by Estrella: it is optimal for the bank to raise capital as expansion prevails, and to absorb losses on capital when recession occurs. Implementing a conservative as a countercyclical buffer could narrow the gap that may occurs at the expansion peak between the risk sensitive requirements, including the buffers, and the optimal bank leverage target, and may help to prevent from the occurrence of a credit crunch when recession prevails. But the bank could also integrate them into its program and raise an even higher target of capital during expansion.

It seems that under a dynamic setting within the bank can adjust its optimal leverage target along the business cycle, the buffer policies help to soften the spill-over of the credit cycle to real sector. But it does not really counteract the risk taking opportunity or restriction that emerges from the cyclical variations into risk sensitive requirements as we depict into our formalization. All depends on the conditions and the costs associated to a raise of capital for the bank.

6. Conclusion

The risk sensitivity of the revised framework brings closer the respective objective of each owner of the bank liability and consequently seems prevent from regulatory trades-off and risk taking incentives embodied into a flat rate capital to asset ratio for the mean-variance ordering asset risk profiles. However the HRHR assets may still offer risk tanking incentives under a risk sensitive regulatory framework.

Actually, under a risk sensitive regulation, regulatory requirements vary with the business cycle, strengthening while economic conditions worsen, softening as the economy gets better. The freeing of regulatory capital during upturns, the "regulatory bonus", can be exploited by the bank to increase its value toward the financing of a riskier asset that enhances the value of bank equities. With the same line of thinking, the "regulatory penalty" that grows as business conditions worsen can restrict the bank risk taking behaviour, pushing it to opt for a lower risk level of asset during recessions. The analysis stresses the procyclicality in terms of asset risk level induced by the risk sensitive regulation, and the resulting bank weakening: potential increase in financial fragility as the expansion prevails.

Although our model embodies limits, the results cannot be excluded under less restrictive assumptions on the bank ownership structure as in a dynamic setting where the bank endogenously determines its optimal leverage ratio. Moreover the novelties of Basel III do not seem to counteract the risk taking opportunity offered by the "regulatory bonus". As in Estrella (2004), our results highlight the crucial role played by the second pillar: the Supervisory Review Process, which encompasses risks that are not taken into the computation of the first pillar (regulatory capital to asset ratio) as some aspects of the bank's risk governance and management structure as a whole, to assess the relevant requirements that can be enforced to the bank. The Four Principles, where the second and the third are reminded below,[13] appear essential to insure a safety banking system:

Principle 2: Supervisors should review and evaluate banks' internal capital adequacy assessments and strategies, as well as their ability to monitor and ensure their compliance with regulatory capital ratios. Supervisors should take appropriate supervisory action if they are not satisfied with the result of this process.
Principle 3: Supervisors should expect banks to operate above the minimum regulatory capital ratios and should have the ability to require banks to hold capital in excess of the minimum.

13. Basel Committee on Banking Supervision (2005) page169 and page 170.

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APPENDIX

Appendix 1.

We have $\Pr\left[\bar{K}+\pi(z)\geq k_r\right]=\alpha_B^*$ which is similar to $\Pr\left[\bar{K}+\pi(z)\leq k_r\right]=\left(1-\alpha_B^*\right)$ or $\Pr\left[\pi(z)\leq k_r-\bar{K}\right]=\left(1-\alpha_B^*\right)$.

Substituting $\pi(z)=S(\bar{p}_i)-p_i(z)\left(S(\bar{p}_i)+\delta+\lambda\right)+\bar{K}(1+\delta)$ in the previous equation we obtain $\Pr\left[\left(S(\bar{p}_i)-p_i(z)\left(S(\bar{p}_i)+\delta+\lambda\right)+\bar{K}(1+\delta)\right)\leq k_r-\bar{K}\right]=\left(1-\alpha_B^*\right)$.

Rearranging we have $\Pr\left[p_i(z)\geq\dfrac{\bar{K}-k_r+S(\bar{p}_i)+\bar{K}(1+\delta)}{S(\bar{p}_i)+\delta+\lambda}\right]=\left(1-\alpha_B^*\right)$ or

$\Pr\left[p_i(z)\leq\dfrac{\bar{K}-k_r+S(\bar{p}_i)+\bar{K}(1+\delta)}{S(\bar{p}_i)+\delta+\lambda}\right]=\alpha_B^*$ which correspond to equation (11) ■

Appendix 2.

We search the value of b such that $\Pr\left[p_i(z)\leq b\right]=\alpha_B^*$ or equivalently $\Pr\left[p_i(z)\leq b\right]=F(b)=\alpha_B^*$.

As the cumulative distribution function of the default rate is given by $F\left(p_i(z)\right)=\phi\left(\dfrac{\sqrt{(1-\rho)}\phi^{-1}\left(p_i(z)\right)-\phi^{-1}\left(\bar{p}_i\right)}{\sqrt{\rho}}\right)$ we have

$$F(b)=\phi\left(\frac{\sqrt{(1-\rho)}\phi^{-1}(b)-\phi^{-1}\left(\bar{p}_i\right)}{\sqrt{\rho}}\right)$$

and b must be solution to $\phi\left(\dfrac{\sqrt{(1-\rho)}\phi^{-1}(b)-\phi^{-1}\left(\bar{p}_i\right)}{\sqrt{\rho}}\right)=\alpha_B^*$ or

$$\frac{\sqrt{(1-\rho)}\phi^{-1}(b)-\phi^{-1}\left(\bar{p}_i\right)}{\sqrt{\rho}}=\phi^{-1}(\alpha_B^*).$$

Rearranging we obtain $\phi^{-1}(b)=\dfrac{\sqrt{\rho}\;\phi^{-1}(\alpha_B^*)+\phi^{-1}\left(\bar{p}_i\right)}{\sqrt{(1-\rho)}}$ and finally

$b=\phi\left(\dfrac{\sqrt{\rho}\;\phi^{-1}(\alpha_B^*)+\phi^{-1}\left(\bar{p}_i\right)}{\sqrt{(1-\rho)}}\right)$ which correspond to equation (12) ■

Appendix 3. Proof of proposition 1.

Recall that the optimal asset class of risk $\bar{p}_i\in\left]0;\bar{p}_{\max}\right]$ chosen at equilibrium by the bank must satisfy the following equation

$$\phi\,(a)\left(S(\bar{p}_i^*)+\delta+\lambda\right)-S(\bar{p}_i^*)=\bar{K}-k_r+\bar{K}(1+\delta) \tag{14}$$

Define $f(\overline{p}_i) = \phi\,(a)\left(S(\overline{p}_i) + \delta + \lambda\right) - S(\overline{p}_i)$, $\phi(a) = \phi\left(\dfrac{\sqrt{\rho}\ \phi^{-1}(\alpha_B^*) + \phi^{-1}(\overline{p}_i)}{\sqrt{(1-\rho)}}\right)$, ϕ being a cumulative distribution function of a standard normal variable, strictly positively and increasing with risk by construction.

According to the HRHR asset assumption, which implies low spreads level for low risk asset, we assume that $\underbrace{\lim}_{\overline{p}_i \to 0} S(\overline{p}_i) \to 0$.

Note also that ρ is computed as in Basel II and it is decreasing with the asset probability of default with $\rho \in [0.12; 0.24]$.

Let's begin by studying the limits of the two parts of equation (14) for $\overline{p}_i \in \left]0; \overline{p}_{\max}\right]$.

1. Limit of $f(\overline{p}_i)$ and $\phi(a)$ when $\overline{p}_i \to 0$.

$\lim\limits_{\overline{p}_i \to 0} f(\overline{p}_i) \to \phi\,(a)(\delta + \lambda)$ as $\lim\limits_{\overline{p}_i \to 0} S(\overline{p}_i) \to 0$.

As $\lim\limits_{\overline{p}_i \to 0} \phi(a) \to \phi\left(\dfrac{\sqrt{\rho}\ \phi^{-1}(\alpha_B^*)}{\sqrt{(1-\rho)}}\right) \to 0$ for $\alpha_B^* \to 1$ we have $\lim\limits_{\overline{p}_i \to 0} f(\overline{p}_i) \to 0$

2. Limit of $\overline{K} - k_r + \overline{K}(1+\delta)$ when $\overline{p}_i \to 0$

The limit of $\overline{K} - k_r + \overline{K}(1+\delta)$ depends on the regulatory capital requirements.

In Basel I: $k_r = \overline{k}_r = 8\ \%$ and $\underbrace{\lim}_{\overline{p}_i \to 0} \overline{K} - \overline{k}_r + \overline{K}(1+\delta) = 2\overline{K}(1+\delta) - \overline{k}_r > 0$ as $\overline{K} \geq \overline{k}_r$. It means that the left part of equation is always constant and positive under risk insensitive capital requirements.

In Basel II: $k_r = k_r(\overline{p}_i)$ with $\dfrac{\partial k_r(\overline{p}_i)}{\partial \overline{p}_i} > 0$ and $\lim\limits_{\overline{p}_i \to 0} k_r(\overline{p}_i) \to 0$. Consequently, we have $\lim\limits_{\overline{p}_i \to 0} \overline{K} - k_r\,(\overline{p}_i) + \overline{K}(1+\delta) = 2\overline{K}(1+\delta) > 0$.

As a consequence, whatever the risk sensitivity of regulatory requirements, $f(\overline{p}_i)$ starts below the left part of equation (14) when the asset belongs to the riskless level class.

3. Limit of $f(\overline{p}_i)$ and $\phi(a)$ when $\overline{p}_i \to \overline{p}_{\max}$.

$\lim\limits_{\overline{p}_i \to \overline{p}_{\max}} f(\overline{p}_i) = \phi\,(a)\left(S(\overline{p}_{\max}) + \delta + \lambda\right) - S(\overline{p}_{\max}) = \phi\,(a)(\delta + \lambda) - S(\overline{p}_{\max})\,(1 - \phi\,(a))$.

as $\lim\limits_{\overline{p}_i \to \overline{p}_{\max}} \phi(a) \to \phi\left(\dfrac{\sqrt{\rho}\ \phi^{-1}(\alpha_B^*) + \phi^{-1}(\overline{p}_{\max})}{\sqrt{(1-\rho)}}\right) \to 1$ we have $\lim\limits_{\overline{p}_i \to \overline{p}_{\max}} f(\overline{p}_i) \to (\delta + \lambda) > 0$.

4. Limit of $\overline{K} - k_r + \overline{K}(1+\delta)$ when $\overline{p}_i \to \overline{p}_{\max}$.

Like before, the limit of $\overline{K} - k_r + \overline{K}(1+\delta)$ depends on the regulatory capital requirement.

In Basel I: $k_r = \overline{k}_r = 8\ \%$ and $\underbrace{\lim}_{\overline{p}_i \to \overline{p}_{\max}} \overline{K} - \overline{k}_r + \overline{K}(1+\delta) = 2\overline{K}(1+\delta) - \overline{k}_r > 0$ as $\overline{K} \geq \overline{k}_r$, which is similar to the previous case.

In Basel II: $k_r = k_r(\overline{p}_i)$ with $\dfrac{\partial k_r(\overline{p}_i)}{\partial \overline{p}_i} > 0$ and $\lim\limits_{\overline{p}_i \to p_{\max}} k_r(\overline{p}_i) \to k_r(\overline{p}_{\max})$. Consequently, we have $\lim\limits_{\overline{p}_i \to p_{\max}} \overline{K} - k_r(\overline{p}_i) + \overline{K}(1+\delta) = 2\overline{K}(1+\delta) - k_r(\overline{p}_{\max}) > 0$ as $\overline{K} \geq k_r(\overline{p}_{\max})$.

As a consequence, we have under Basel I $\overline{K} - \overline{k}_r + \overline{K}(1+\delta) < (\delta + \lambda)$ i.f.f. $\overline{K} < \dfrac{(\delta+\lambda) + \overline{k}_r}{2(1+\delta)}$. Similarly, under Basel II we have $2\overline{K}(1+\delta) - k_r(\overline{p}_{\max}) < (\delta+\lambda)$ i.i.f. $\overline{K} < \dfrac{(\delta+\lambda) + k_r(\overline{p}_{\max})}{2(1+\delta)}$.

These two conditions give rise to an unique one $\overline{K} < \dfrac{(\delta+\lambda) + k_r}{2(1+\delta)}$, with $k_r = \overline{k}_r$ in the case of Basel I and $k_r = k_r(\overline{p}_{\max})$ in Basel II.

Finally, the nature of the optimal equilibrium depends on the sensitivity of the spread to the risk of the asset.

Firstly, assume that $\dfrac{\partial S(\overline{p}_i)}{\partial \overline{p}} < \dfrac{\partial \phi(a)}{\partial \overline{p}} \cdot \dfrac{S(\overline{p}_i) + \delta + \lambda}{1 - \overline{p}_i}$ which leads to $\dfrac{\partial f(\overline{p}_i)}{\partial \overline{p}} > 0$. As $f(\overline{p}_i)$ is continuously increasing on $\overline{p}_i \in]0; \overline{p}_{\max}]$ two cases are possible.

If $\overline{K} < \dfrac{(\delta+\lambda) + k_r}{2(1+\delta)}$, we have $f(0) < \overline{K} - k_r + \overline{K}(1+\delta)$ and $f(\overline{p}_{\max}) > \overline{K} - k_r + \overline{K}(1+\delta)$ with $\dfrac{\partial f(\overline{p}_i)}{\partial \overline{p}} > 0$. Consequently, there is an interior solution $\overline{p}_i^* \in]0; \overline{p}_{\max}]$ such that $f(\overline{p}_i^*) = \overline{K} - k_r + \overline{K}(1+\delta)$.

If $\overline{K} \geq \dfrac{(\delta+\lambda) + k_r}{2(1+\delta)}$, we have $f(\overline{p}_i) < \overline{K} - k_r + \overline{K}(1+\delta), \forall \overline{p}_i$. As the left part of equation is always higher than the right part, the class of risk retains by the bank would be the riskiest one available $\overline{p}_{\max}$.

Secondly, assume $\dfrac{\partial S(\overline{p}_i)}{\partial \overline{p}_i} > \dfrac{\partial \phi(a)}{\partial \overline{p}_i} \cdot \dfrac{S(\overline{p}_i) + \delta + \lambda}{1 - \overline{p}_i}$ which leads to $\dfrac{\partial f(\overline{p}_i)}{\partial \overline{p}_i} < 0$. As $f(\overline{p}_i)$ is continuously decreasing on $\overline{p}_i \in]0; \overline{p}_{\max}]$ and $f(0) < \overline{K} - k_r + \overline{K}(1+\delta)$, the left part of equation is always higher than the right part, the class of risk retains by the bank would be the riskiest one available $\overline{p}_{\max}$.

The proof of proposition 1 is completed ■