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Human Capital Dispersion and Incentives to Innovate

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Abstract

Do policies that alter the allocation of human capital across individuals affect the innovation capacity of an economy? To answer this question, I extend Romer's (1990) growth model to allow for individual heterogeneity. I find that the value of an invention rises with equality. If skills and talents are evenly distributed, inventions are more widely adopted in production and users are willing to bid a higher price. Therefore, more equality is associated with a larger share of the population employed in the business of invention. However, inventors of an equal society are not as creative as those of an unequal one. As a result an inverted-U curve relating inequality and the innovation rate emerges, indicating that departures from extreme forms of equality or inequality are growth-enhancing. I discuss evidence that agrees with the main implications of the analysis, namely that the market size and the number of inventors are negatively affected by inequality. Finally, a calibration exercise suggests that in recent decades the U.S. has been in the ascending portion of the inequality-growth curve.

Keywords: human capital, inequality, innovation

JEL Classification: O15; O31; O41; H52; J24.

1 Introduction

Reforms of the education system are often dictated by the desire to foster students' cognitive abilities or by shifts in the notions of equality of opportunities and social justice. For instance, some countries choose to track students into different school types, hierarchically structured by performance, as early as the age of ten (this is the case in Austria, Germany, Hungary, and the Slovak Republic), whereas others keep the entire lower secondary school system

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comprehensive, or design some blend of the two systems. In an international comparative study, Hanushek and Wößmann (2006) found that the choice of the structure of the education system affects the distribution of human capital. In particular, their study suggests that the variance of pupils' attainments is greater in a tracking than in a comprehensive system. Another educational policy that affects, perhaps more directly, the dispersion of human capital is the allocation of government funding across different levels of education.[1]

Policy makers seem to be quite attentive to news reporting an increased gap in students' achievement. In 1982, an official inquiry by the Cockcroft Committee in England and Wales found that a seven year difference existed in pupils' mathematical attainments. This conclusion had fundamental consequences in the reorganization of the British school system. A National Curriculum established at the end of the 1980s set the target of containing 11-year-old pupils' attainment within less than six years of difference for 80 percent of the pupils.[2] The 2001 No Child Left Behind Act echoes similar concerns. It set the year 2014 as the deadline by which schools were to close the test-score gaps between minorities and white students.

What are the long-run consequences of policies that reduce human capital inequality? I propose to answer this question by assessing how a country's ability to innovate is linked to the shape of the human capital distribution.

I will argue that any distributional change in human capital puts into motion forces affecting unavoidably both the demand and the supply of inventions. To gain some insights into the reasoning that will be developed in this study, imagine an economy in which inventions are supplied by highly educated or talented individuals, whose earnings depend on their productivity and on the market value of their inventions. Final good producers – who constitute the remaining part of the population – demand a variety of capital goods that are built on the basis of designs created by the inventors. The larger the scale of utilization of a given type of capital good, the greater the social and private value of its design, for this is a non-rival good that can be replicated at no cost. Under the assumption that producers are subject to the law of diminishing returns on capital – intended in a broad sense to include both human and physical capital – the demand for capital goods, and consequently the value of an invention, declines as inequality increases. This result is derived by applying the Jensen's inequality to a concave production function whereby it is shown that aggregate output is larger when inputs are used in the same (average) quantity by each producer than when they are unevenly distributed across producers. It follows that the inventors's payoff out of each invention diminishes as inequality rises, a circumstance that induces a number

[1] Castelló and Doménech (2002) document an historical cross-country convergence of this type of educational inequality.

[2] For a detailed discussion see S. Prais, 1993

of them to switch from the business of creation to that of final output production. But a reduced number of inventors does not necessarily mean a lower innovation rate, for this depends on the inventors's ingenuity as much as on their number. A second assumption of this study is that a society in which the distribution of human capital is more widely dispersed is more likely to have more creative inventors than one populated by individuals whose abilities are closer to those of the average individual. Therefore, more inequality improves inventors's creativity but also limits the size of the market. One important insight that results from the analysis is that the creativity-effect dominates (is dominated by) the market-size effect for relatively low (high) level of inequality. In other words, an inverted-U relationship between inequality and the innovation rate emerges, indicating that any movement from extreme forms of equality or inequality is growth-enhancing.

The analysis is developed within the tradition of ideas-based models, as exemplified in Romer (1990) that I extend here to allow for individual heterogeneity. The growth literature has studied extensively the dynamic consequences of human capital formation. Lucas (1988) and Uzawa (1965) have hypothesized that the growth rate of the economy is driven by the accumulation of human capital, whereas Nelson and Phelps (1966) and Romer link such a rate to the stock of human capital, rather than its expansion over time. However, this early literature has not addressed the question of how the distribution of human resources across different economic activities affects subsequent growth. Baumol (1990), marshalling a great variety of historical evidence, argued, convincenly, that the distribution of entrepreneurial talents into different economic activities is the main key in understanding the rise or fall of societies in the long run. A similar point was made by Murphy, Shleifer, and Vishny (1991). These two studies imply that linking the long run growth of the economy only to the overall supply of human capital, as it is done in the endogenous growth models, is unlikely to account for the remarkable success of capitalist economies.

A more detailed review of the literature is given in Section (2). This is followed by two examples that show why schooling can be seen as a redistributive tool. Section (4) describes the extension of Romer's model, and proves the existence of an equilibrium for a generic human capital distribution. The aggregation and the determination of the balanced growth path are carried out in section (5). Section (6) illustrates the links between inequality and the innovation rate. Section (7) suggests a way to calibrate the model to the U.S. economy. Empirical evidence in support of the main implications of the theory is discussed in Section (8). Section (9) concludes.

2 Review of the Literature

The extensive literature on education that investigates the learning and social implications of alternative educational system is germane to the motivation of this work. One issue that seems particularly debated in these studies is the trade-off between the mean and variance in knowledge acquisition (see, among others, Argys, Rees, and Brewer (1996), Dobbelsteen, Levin and Oosterbeek (2002), Figlio and Page (2002), Hanushek *et al.* (2003), Meier(2004), and Betts and Shkolnik (2000)). Heath (1984) and Slavin (1990) review studies investigating this issue with a focus on the UK and on the U.S. school system respectively, and Meghir and Palme (2005) discuss the effects of educational reforms in Sweden and other European countries. Herrnstein and Murray (1996, ch. 4) contend that the increased variance in education is partly due to the introduction of sophisticated procedures that track students by ability as they advance to the highest level of education.[3]

This paper is also related to the theoretical and empirical literature that has explored the links between inequality and growth. Four excellent papers surveying these relationships are Benabou (1996b), Aghion, Caroli, and Garcia-Peñalosa (1999), Fernandez (2001) and Benabou (2004). The starting point of the debate is the surprising evidence found by Perotti (1996), Alesina and Rodrik (1994), and Persson and Tabellini (1994), that equality and growth may go hand in hand. These studies spurred numerous works searching for explanations. Benabou (1996a), Durlauf (1996), Aghion and Bolton (1997), Piketty (1997), Benabou (2002), Galor and Moav (2004), building or extending previous works by Loury (1981), Evans and Jovanovic (1989), Banerjee and Newman (1993), and Galor and Zeira (1993), entertain the hypothesis that credit constraints limit the ability to invest in physical or human capital for people with little or no endowments. Similarly, I find that the value of an idea, which can be thought as the result of human capital investment, declines with inequality, and that such decline is due to the presence diminishing returns on capital. But my argument leads to the conclusion that an unequal economy may actually grow faster than an equal one, for it is populated by more productive innovators. A similar type of reasoning is developed by Galor and Tsiddon (1997) in which knowledge acquired by a selected group of people generates a global externality that favors human capital accumulation.

This paper is also close to Zweimüller (2000) and to Föllmi and Zweimüller (2006) in the way innovation, market size, and inequality are related. They conjecture that redistribution affects the market size because individuals differ in their preferences or in their timing of

[3]Recently the press documented that an increasing fraction of both federal and state US financial aids to students is merit-based. (June Kronholz, the Wall Street Journal (Eastern edition). New York, N.Y.: Sep 23, 2002. p. B.1)). The beginning of this new trend is usually associated with the legislation passed by the state of Georgia in 1993 when it launched the first state merit program, known as the HOPE scholarship, an attempt to reduce the flow of bright students to out-of-state colleges, and increase college attendance.

adoption of new technologies. Here individuals have identical preferences and there is no delay in adopting an innovation. Instead redistribution (of human capital) affects the market size in a same way as in credit-constraint literature just discussed. There is a long stream of research following the influential contribution of Schmookler (1966) that associates the level of demand with the rate of technological progress (see, among others, Krugman (1993), Murphy, Sheifer, and Vishny, (1989a, 1989b), and Sørensen (1999)). It will be shown that the innovation rate responds very differently to a given change in the market size, depending on whether it was caused by inequality or by technological developments.

This paper also offers an interpretation to the more recent empirical evidence on the association between growth and inequality. Forbes (2000) questions Perotti's (1996) estimation and found that an increase of the Gini coefficient in 0.1 is paralleled by a rise in the average annual growth rate in the subsequent 5-year period of 1.3 percent. Barro (2000), following a different methodology, found a more modest positive correlation of 0.5 between growth and inequality for high- and medium-income countries and a negative one of about the same magnitude for low-income countries. Castelló and Doménech (2002) found a negative relationship when inequality is computed on educational capital. Banerjee and Duflo (2003) attribute the conflicting conclusions reached by the empirical literature to the linear structure of the estimation. They contend that since the actual relationship is not linear – a point that agrees with this paper – different variants of a linear specification are likely to deliver a different sign for the estimated coefficient. Voitchovsky (2005) suggests that the ambiguity may be due to the custom of using Gini index to measure inequality. She found that growth is positively associated with top income inequality, but the relationship turns out to be negative with bottom income inequality.

3 Examples of Policies that Alter the Dispersion of Human Capital

Traditionally the economic growth literature has seen schooling as a mean to accumulate human capital. In this study I consider its redistributive aspects instead. A cursory reading of the news on education reveals that nearly every day policy educators are confronted with choices that involve the distribution of human resources within a school or across schools. For instance, public schools have to choose whether or not to have a program for 'gifted' students.[4] Superintendents or state legislators have to decide whether the teachers' reward

[4] Samuel G. Freeman reports in an article published in the *New York Times* on November 22, 2006, that Ms. Rohloff, a principal of a New York City public school, decided to shut down the honors program because it was too expensive to operate.

system should devised so that best teachers end up in schools attended by kids with high needs or by kids from well-off families.[5] And in countries where the education system is centralized and education is publicly provided usually the government, or the educational institutions themselves, set a quota on how many students are admitted on each subject.

The list of educational policies that affect directly or indirectly the variance of human capital is endless. Nevertheless, the redistributive consequences of alternative arrangements can be illustrated with two simple examples that capture the tension of the debate on educational reforms. Consider a society in which half of the population is born with a level of ability (or talent) $A = 1$, and the other half with $A = \Omega > 1$. The mean and the variance of the distribution over A are $\mu_A = \frac{1+\Omega}{2}$ and $\sigma_A^2 = \frac{(\Omega-1)^2}{4}$, respectively.

3.1 Example 1

In one hypothetical situation students learn from each other. A student with ability A_S attending a school with classmates of average ability $\bar{A}_S$, acquires a level of human capital

$$H(A_S, \bar{A}_S) = \lambda[\gamma A_S + (1-\gamma)\bar{A}_S],$$

where $\lambda > 1$ is a learning parameter, and $\gamma \in [0,1]$ weighs the importance of a student's ability relative to peer-learning externalities. The difference between a comprehensive and an ability-grouping system can be captured by contrasting two kinds of matching. In one, classes are homogenous with respect to abilities. Therefore, the average level of human capital is $\lambda\mu_A$ and the variance $\lambda^2\sigma_A^2$. In the other, every class has an equal number of high and low ability students. In this case the mean is still $\lambda\mu_A$ but the variance is lower: $\gamma^2\lambda^2\sigma_A^2$. This learning function is very simple, and yet its main implications are in line with Hanushek and Wößmann (2006) cross-country study in which it is shown that educational achievements are more dispersed in an ability-grouping system than in a comprehensive one, and that the average scores are not statistically different in the two arrangements.

3.2 Example 2

In a second situation students' learning depends on their innate abilities and those of their teachers. Imagine for instance that a fraction η of each of the two groups of individuals plays the role of 'teachers' and the remaining fraction plays that of 'students' (the parameter η

[5] Larry Abramson reported on National Public Radio in the Morning Edition of November 23, 2006, that superintendents in North Carolina are offering annual bonuses of up to $15000 to good teachers who are willing to work in targeted schools. In the report it is also stated that "[s]uperintendents around the country have long insisted they need to be able to transfer the best teachers to troubled schools." Apparently, in many states union contracts do not allow that kind of action.

may be thought as one that sets the optimal combination between students and teacher). By attending schools both students and teachers acquire knowledge, H. The learning technology for a student with ability A_S who is matched with a teacher with ability A_T is

$$H(A_S, A_T) = \lambda[\gamma A_S + (1-\gamma)A_T],$$

where the parameters λ and γ are to be interpreted in a similar way as those used in Example 1. The teacher's learning function is

$$H(A_T, \bar{A}_S) = \lambda[\gamma A_T + (1-\gamma)\bar{A}_S],$$

where $\bar{A}_S$ is the average ability of the students with whom the teacher has been matched. Again, I compare two kinds of arrangements. In one, high-(low-)ability students are matched with high-(low-)ability teachers. In the other the matching goes the other way around. In either case the mean of H is $\lambda\mu_A$. However, the variance differs. When good teachers work with high ability students the variance is $\lambda^2\sigma_A^2 = \lambda^2\frac{(\Omega-1)^2}{4}$, whereas when good teachers are matched with low-ability students the variance is $\lambda^2(1-2\gamma)^2\frac{(\Omega-1)^2}{4}$ that is smaller than $\lambda^2\frac{(\Omega-1)^2}{4}$ for $0 < \gamma < 1$.

3.3 Evidence

The learning parameter λ depends on the years of education. Therefore, regardless of the educational system adopted both the variance and the mean tend to increase as students advance in grades. This intuition is confirmed by casual observation. Table (1) shows the test scores in mathematics of four ethnic groups computed in a recent survey of about 100,000 first-grade students conducted in various U.S. schools. It appears that the standard deviation and the mean increase for all four ethnic groups from fourth to seventh grade.[6] Still two groups can have the same mean and a different dispersion, as it is for instance the case of African American and Hispanics in forth grade. In addition, the table shows that the Non-Minority group has the larger mean and yet the smaller standard deviation. It is easy to come across a similar kind of evidence in cross-country comparisons. Table (2) shows the pupils' achievements in mathematics at the age of 15, recorded in a recent survey conducted by the Programme for International Student Assessment (PISA) in two pairs of countries. Each pair shows a similar mean by a different variance.

[6] There is a very high correlation between the changes in standard deviations and means, as suggested by the examples. Notice however that the standard deviation increases about three times more than the mean. Perhaps the pattern reflectes a more pronounced effect of sorting as kids advance in grade.

3.4 Meritocracy

How is the choice of the school system linked with inequality? Benabou (2000) formalizes the concept of inequality by introducing a two-dimensional measure of meritocracy that captures the concepts of inequality in opportunities and inequality in outcomes. Meritocracy in opportunities measures the extent to which background factors, such as race, gender, or religion, introduces a wedge between the distribution of earnings and that of talents or abilities. In this paper I do not consider issues of discrimination, therefore I assume that there is meritocracy in opportunities. Meritocracy in outcomes is instead intended as the absolute reward of talent. Next I show that alterations in the educational arrangements affect meritocracy in a similar way as fiscal policy does. Assume that income is determined according to

$$Y = zH,$$

where z is technological index, and H is human capital. Let meritocracy in outcomes, m, be the ratio between the variance of disposable income and that of abilities. Then

$$m = \frac{[z(1-\tau)]^2 \sigma_H^2}{\sigma_A^2},$$

where τ is the tax rate. In Example 1 m is equal to $[z\lambda(1-\tau)]^2$ in the ability-grouping case and to $[z\gamma\lambda(1-\tau)]^2$ in the mixed-grouping case. In Example 2, the values of m in the corresponding two cases are $[z\lambda(1-\tau)]^2$ and $[z\lambda(1-2\gamma)(1-\tau)]^2$, respectively. Clearly, in both examples going from an ability-grouping to a mixed-grouping one leads to a reduction in m (as long as $0 < \gamma < 1$) in the same way as an increase in the tax rate would. Therefore, in principle, schooling affects the measure of meritocracy in outcomes as much as fiscal policy does.

4 The Basic Model

The model that I propose is an extension of the well known Romer's R&D growth model (1990), in which innovation activities are carried out by profit maximizing individuals and come in the form of an expansion in the variety of capital goods employed for the production of a final good. In Romer (1990) all individuals are endowed with the same amount of human capital. The objective of this section is to study the consequences of relaxing this assumption.

The economy is populated by infinitely-lived individuals of measure 1. Each individual is endowed with one unit of time and with some human capital h that are used either to produce a final good or to generate inventions. The flow of final good produced by an individual with human capital h that employs N durable goods is

$$y(h) = zh^\alpha \int_0^N q(i)^\beta di, \tag{1}$$

where $q(i)$ denotes the quantity of durable good $i \in [0, N]$, and where α, β, and z are positive parameters.

The chief advantage of specifying the output function as in Eq. (1) is that the marginal productivity of durable good i does not depend on that of durable good j (for $i \neq j$), a feature that greatly simplifies the derivation of the demand function for durable goods. This production function is similar to the one proposed by Ethier (1982), Spence (1976), Dixit and Stiglitz (1977), and Romer (1990), except that here each individual employed in production runs his own firm[7] and hence is the residual claimant of firm's profits.

Profit maximization leads to the optimal condition

$$zh^\alpha \beta q(i)^{\beta-1} = p(i), \tag{2}$$

where $p(i)$ is the price of one unit of intermediate input i. The previous expression can also be written as a (direct) demand function:

$$q(i) = (\frac{z\beta}{p(i)})^{1/(1-\beta)}(h)^{\alpha/(1-\beta)}. \tag{3}$$

4.1 The monopoly price

The market demand function for durable good i is derived by summing-up all individual demand curves. Integrating equation (3) with respect to h, we get

$$Q(i) = (\frac{z\beta}{p(i)})^{1/(1-\beta)} \int_0^{+\infty} (h)^{\alpha/(1-\beta)} d\tilde{F}(h), \tag{4}$$

where $\tilde{F}(h)$ denotes the number of final good producers with a level of human capital equal or less than h. One unit of durable good i is obtained from η units of forgone consumption. If this is rented at rate r, the flow of marginal cost of producing one unit of durable good i is the interest payment $r\eta$. It is easy to verify that the non-discriminatory monopoly rental price is the same for all durable goods, that is $p(i) = \bar{p} = r\eta/\beta$. Therefore, a design generates the flow of rent at time s

$$\pi(s) = (1-\beta)(z\beta)^{\frac{1}{(1-\beta)}}(\frac{\beta}{r(s)\eta})^{\frac{\beta}{(1-\beta)}} \int_0^\infty h^{\alpha/(1-\beta)} d\tilde{F}(h). \tag{5}$$

[7] The departure is dictated by the desire of relating the analytical results of this paper to the literature that studies the links between income growth and inequality under the assumption that agents have a limited access to the credit market.

The design's value is given by the discounted stream of profits that the intermediate good producer expects to gain from renting the durable good to a final good producer, namely

$$P(t) = \int_t^{t+T} \pi(s) e^{-R(t,s)} ds,$$

where T is the length of time during which the inventor extracts a rent from an invention, $R(t,s) = \int_t^s r(v)dv$, and $r(v)$ is the instantaneous interest rate at time v. If this is constant (a circumstance that will be verified in the equilibrium described below), the value of an invention can be written as

$$P = \Delta \int_0^{\infty} h^{\alpha/(1-\beta)} d\tilde{F}(h). \tag{6}$$

where $\Delta = \frac{1}{r}(1 - e^{-rT})(1-\beta)(z\beta)^{1/(1-\beta)}(\frac{\beta}{r\eta})^{\beta/(1-\beta)}$. In order to determine the allocation of individuals between the final-good sector and the innovation sector, a reward function for the inventors must be specified.

4.2 Labor market equilibrium

Someone with human capital h can produce a flow of inventions equal to $\delta h^{\phi} N$, where δ is a productivity parameter and N is the number of existing inventions (these provide useful knowledge in elaborating a new design). The parameter $\phi > 0$ is the elasticity of the flow of ideas with respect to human capital. Thus the flow of income for an inventor with human capital h is the flow of ideas multiplied by their unit price:

$$w_I(h) = \delta P N h^{\phi}. \tag{7}$$

The final good producer chooses an amount of durable goods according to equation (3) with $p(i)$ being replaced by $r\eta/\beta$, if the design still commands a monopoly rent, and by the marginal cost $r\eta$ otherwise. Let $\bar{q}^m(h)$ and $\bar{q}^c(h)$ be the resulting demand function under the two market structures, respectively. Let C and M measure the number of old vintages rented at competitive and monopoly price respectively ($N = C + M$). It follows that the flow of output produced by an individual with skills h employed in the final good sector is

$$y(h) = zh^{\alpha}[C(\bar{q}^c(h))^{\beta} + (M)(\bar{q}^m(h))^{\beta}].$$

This expression can be rearranged as

$$y(h) = z^{\frac{1}{1-\beta}} h^{\frac{\alpha}{1-\beta}} (\frac{\beta}{r\eta})^{\beta/(1-\beta)}[c + (1-c)(\beta)^{\beta/(1-\beta)}]N, \tag{8}$$

where $c = C/N$. One can show that the income of an individual h employed in the final good sector, denoted with $w_y(h)$, is simply $(1-\beta)y(h)$, that is

$$w_y(h) = \chi N h^{\frac{\alpha}{1-\beta}}, \tag{9}$$

where $\chi = (1-\beta)z^{1/(1-\beta)}(\frac{\beta}{r\eta})^{\beta/(1-\beta)}[c+(\beta)^{\beta/(1-\beta)}(1-c)]$. At this point all the elements needed to state a condition that indicates how people sort themselves between the occupation of inventors and that of final-good producers have been discussed. Since such condition turns out to depend crucially on the relationship between ϕ and $\frac{\alpha}{1-\beta}$, I impose the following restriction:

(A1) The elasticity parameter ϕ is larger than the ratio $\frac{\alpha}{1-\beta}$.

Lemma 1 *Under assumption (A1), any inventor has a level of human capital that is at least as large as that of any final good producer.*

Proof. Let an individual with human capital $\bar{h}$ be indifferent between being employed as an inventor or as a final good producer. Then Eqs. (7) and (9) imply that $\bar{h}$ must satisfy the condition

$$\delta \bar{h}^{\phi} \bar{P} = \bar{\chi} \bar{h}^{\frac{\alpha}{1-\beta}}, \tag{10}$$

where a bar on P and χ indicates the value of these variables when $h = \bar{h}$. An individual with human capital h earns $\delta \bar{P} N h^{\phi}$ as an inventor and $\bar{\chi} N h^{\frac{\alpha}{1-\beta}}$ as a producer. If $\phi > \frac{\alpha}{1-\beta}$ the ratio $\frac{\delta \bar{P} h^{\phi}}{\bar{\chi} h^{\frac{\alpha}{1-\beta}}}$ is larger (smaller) than one whenever $h > (<)\ \bar{h}$. Hence the claim ■

Remark A1 is only a sufficient condition. It can be verified that the claim holds also for $\phi < \frac{\alpha}{1-\beta}$ as long as $\phi > \frac{\alpha}{1-\beta} - \frac{\bar{h}}{\bar{P}} \frac{\partial \bar{P}}{\partial \bar{h}}$.

Combining Eq. (6) with condition (10) we get the key equation of the model:

$$\underbrace{\bar{h}^{\phi} \delta \Delta \int_0^{\bar{h}} h^{\alpha/(1-\beta)} dF(h)}_{\text{Price of a design}/N} = \bar{\chi} \bar{h}^{\frac{\alpha}{1-\beta}}. \tag{11}$$

Notice that the threshold $\bar{h}$ is not affected by the technological parameters η or z and that it is constant over time if r and $F(.)$ are also time invariant. For a given $\bar{h}$ the growth of ideas, g_N, is obtained by taking the ratio between inventors' flow of designs and the stock of existing designs:

$$\bar{g}_N \equiv \frac{\dot{N}}{N} = \delta \int_{\bar{h}}^{+\infty} h^{\phi} dF(h). \tag{12}$$

It is useful to observe that if $\bar{g}_N$ has been constant in the past T years, the share of intermediate goods traded in a monopolistic competition market, $1-c$, is equal to $\exp(-\bar{g}_N T)$ and the share sold at a perfectly competitive price is $1-\exp(-\bar{g}_N T)$. In this case

$$\bar{\chi} = (1-\beta) z^{1/(1-\beta)} (\frac{\beta}{r\eta})^{\beta/(1-\beta)} [1 - (1-(\beta)^{\beta/(1-\beta)}) \exp(-\bar{g}_N T)]. \tag{13}$$

Proposition 1 *Under assumption (A1) there exists an $\bar{h}$ that solves the system of equation (11)-(13).*[8]

Proof. We need to prove that $\bar{h}$ solves the equation

$$\delta\Delta \int_0^{\bar{h}} h^{\alpha/(1-\beta)} dF(h) = \bar{\chi}\bar{h}^{\frac{\alpha}{1-\beta}-\phi}.$$

Let the functions $g(\hat{h})$, $\chi(\hat{h})$ and $P(\hat{h})$, denote the right-hand side of Eqs (12) and (13), and the left-hand side of the above equation where the variable $\bar{h}$ is replaced by $\hat{h}$. $P(\hat{h})$ is an increasing function. As for the right-hand side of the above equation notice that in virtue of (A1) $\hat{h}^{\frac{\alpha}{1-\beta}-\phi}$ is a decreasing function that goes to zero (infinity) as $\hat{h}$ goes to infinity (zero). Finally, it is easy to verify that $\chi(\hat{h})$ is also a decreasing function bounded from above and below. Hence the claim. ■

5 Aggregation and Balanced Growth Path

5.1 Production Side

The aggregate stock of capital is computed by summing-up all the intermediate goods in use and multiplying the resulting quantity by η – the quantity of consumption goods required to build one unit of capital. Eq. (4) implies that demand for one type of intermediate goods is $(\frac{z\beta}{p(i)})^{1/(1-\beta)} \int_0^{\bar{h}} (h)^{\alpha/(1-\beta)} dF(h)$. Since there are N intermediate goods, of which C are priced at the marginal cost ($p = r\eta$) and the remaining $N - C$ at the monopoly price $r\eta/\beta$, aggregate capital is

$$K = \eta(\frac{z\beta}{r\eta})^{1/(1-\beta)} N[c + (1-c)(\beta)^{1/(1-\beta)}] \int_0^{\bar{h}} h^{\alpha/(1-\beta)} dF(h). \tag{14}$$

Similarly, final output is calculated by using equation Eq. (8):

$$Y = z^{\frac{1}{1-\beta}}(\frac{\beta}{r\eta})^{\beta/(1-\beta)} N[c + (1-c)(\beta)^{\beta/(1-\beta)}] \int_0^{\bar{h}} h^{\frac{\alpha}{1-\beta}} dF(h). \tag{15}$$

Notice that if $\bar{h}, r$, and c are constant $Y = AK$, where $A = \frac{r}{\beta}\frac{c+(1-c)(\beta)^{\beta/(1-\beta)}}{c+(1-c)(\beta)^{1/(1-\beta)}}$.

[8] In principle one can determine $\bar{h}$ under the more general condition that $\frac{\alpha}{1-\beta} \neq \phi$. However, I focus the attention on the case in which (A1) applies, as such a restriction will be imposed on the equilibrium to be discussed in the coming section.

5.2 Saving

To close the model, the consumer preferences need to be specified. I assume a utility function with constant elasticity of intertemporal substitution

$$\int_0^\infty \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt,$$

that implies that the intertemporal optimization condition for a consumer faced with the an interest rate $r(t)$ is

$$g_c(t) = \frac{1}{\sigma}(r(t) - \rho), \tag{16}$$

where $g_c(t)$ is the annual growth rate of per capita consumption.

5.3 Balanced Growth Path

I want to characterize an equilibrium in which the variables K, Y, and N, grow at constant exponential rates, and in which $\bar{h}$, $\bar{g}_N$, and the interest rate are constant. For a given interest rate, Eqs. (11)- (13) pin down the equilibrium value $\bar{h}$. These will not change over time as long as $F(.)$ is time-invariant. By inspecting Eqs. (14) and (15) one realizes that if the interest rate is constant both output and capital growth at rate $\bar{g}_N$ as well. The market clearing condition $Y = C + \dot{K}$ implies that aggregate consumption also grows at the same rate than K and Y, that is $\bar{g}_N$. Since population is constant this is also the rate at which per capita consumption grows. Finally, the constant interest rate is obtained by equating $\bar{g}_N$ and g_c.

6 Discussion

To gain some insights into the link between inequality and innovation implied by the model, I compare two special cases: In one all individuals are identical as in Romer (1990). In a second scenario there are two types of individuals one with a high and the other with a low level of human capital. For simplicity, in both illustrations the interest rate is exogenously given.[9]

[9] The calibration exercise in section (7) includes the general equilibrium effects related to movements in the interest rate.

6.1 Romer as a special case

Consider a degenerate frequency distribution $f(h)$ with a mass of probability one at $h = H$, and let $T \to +\infty$. The equilibrium equation in Eq. (11) implies that

$$\frac{1}{r}H^{\frac{\alpha}{1-\beta}}(1-l) = \frac{1}{\delta\beta}(H)^{\frac{\alpha}{1-\beta}-\phi}, \tag{17}$$

where l is the number of inventors. The previous equation leads to

$$l = 1 - \frac{r}{\delta\beta H^{\phi}}. \tag{18}$$

Following Eq. (12) the rate of innovation is $g_N = \delta l H^{\phi}$ that combined with the previous expression yields

$$g_N = \delta H^{\phi} - \frac{r}{\beta} \tag{19}$$

that corresponds to the key equation in Romer (1990) – that is equation (11') at p. S92– provided that $\phi = 1$.

6.2 Market Size and Research Productivity

The population is equally split between two types of individuals, denoted as type-1 and type-2, whose human capital is H_1 and H_2, respectively, where $H_1 = H(1-\varepsilon)$, $H_2 = H(1+\varepsilon)$ with $\epsilon > 0$. Let $l_1$and l_2 be the fraction of type-1 and type-2 individuals who are inventors. From Lemma 1 we know that the most talented individuals are inventors. If this group is less than half of the population, then $l_2 < 1$ and $l_1 = 0$. In this case, the labor market equilibrium condition is

$$\frac{1}{r}[\frac{1}{2}H_1^{\frac{\alpha}{1-\beta}} + \frac{1}{2}H_2^{\frac{\alpha}{1-\beta}}(1-l_2)] = \frac{1}{\delta\beta}(H_2)^{\frac{\alpha}{1-\beta}-\phi}. \tag{20}$$

Alternatively, if $l_2 = 1$ and $l_1 > 0$ the free entry condition becomes

$$\frac{1}{r}\frac{1}{2}H_1^{\frac{\alpha}{1-\beta}}(1-l_1) = \frac{1}{\delta\beta}(H_1)^{\frac{\alpha}{1-\beta}-\phi}. \tag{21}$$

Since the number of inventors is likely to be less than half of the population, I continue the exposition based on Eq. (20). To separate the competing effects of market size and research productivity, I rearrange this equation as

$$\frac{1}{2}l_2 = \frac{1}{2}[1 + \underbrace{(\frac{1-\epsilon}{1+\epsilon})^{\frac{\alpha}{1-\beta}}}_{\text{Market-Size}}] - \underbrace{\frac{r}{\delta\beta H^{\phi}(1+\epsilon)^{\phi}}}_{\text{Productivity}}. \tag{22}$$

The rise in inequality is captured by an increase in ϵ. As this goes up, the demand for intermediate products, and with it the design's value, declines – see the term inside the square bracket – whereas the inventor's productivity rises. Since an inventor generates more designs but earn less out of each one sold, his or her reward – and therefore the number of individuals that choose to innovate – may go up or down as ϵ increases, depending on which one of the two effects is larger. In Fig. (1.A), where $\frac{1}{2}l_2$ is plotted against ϵ, the productivity effect is more important than the decline in the invention's value for very small departures from an equal situation (small ϵ). But as the economy becomes more and more unequal, the decline in the design's price becomes the dominant force and consequently the number of innovators diminishes. Therefore an inverted-U shape may emerge.

To see more in details why inequality causes a reduction in the price of an invention through the market size it is useful to rearrange Eq. (6) as:

$$P = \Delta H^{\frac{\alpha}{1-\beta}}[\underbrace{\frac{1}{2}(1-\epsilon)^{\frac{\alpha}{1-\beta}} + \frac{1}{2}(1+\epsilon)^{\frac{\alpha}{1-\beta}}}_{\text{inequality}} - l_2\frac{1}{2}(1+\epsilon)^{\frac{\alpha}{1-\beta}}]. \tag{23}$$

The first two terms inside the square brackets capture the negative effect of inequality – total output declines with inequality, and therefore the rent of the inventor goes down as well. The term $\frac{1}{2}l_2$, has a negative sign because for each new inventor there is one fewer producer. In other words the greater R&D sector the fewer the technology adopters, and the lower the invention's value. But this is only a second-order effect that just mitigates the decline of P. Indeed, by combining Eqs. (22) and (23) one obtains $P = \frac{\Delta r}{\delta\beta}[H(1+\epsilon)]^{\frac{\alpha}{1-\beta}-\phi}$ indicating that the variation in l_2 is not strong enough to counterbalance the direct effect of inequality on the market size. (See panel B that plots P against ϵ.)

Next I turn to the growth rate of innovation that is given by

$$g = \delta H_2^{\phi}\frac{1}{2}l_2.$$

Because $\frac{1}{2}l_2$ declines with ϵ when this is large enough and H_2 always increases with it, the innovation rate has a non-monotonic relationship with ϵ. To understand how the relative strength of the two competing forces are affected by technological parameters, it is useful to rearrange the above expression as

$$g_N = \delta H^{\phi}(1+\epsilon)^{\phi}\frac{1}{2}[1 + (\frac{1-\epsilon}{1+\epsilon})^{\frac{\alpha}{1-\beta}}] - \frac{r}{\beta}. \tag{24}$$

If there are constant returns on capital ($\frac{\alpha}{1-\beta} = 1$), a wider dispersion in human capital has always a positive effect on the rate of innovation under the assumption (A1). But if $\frac{\alpha}{1-\beta} < 1$, the advantage of having a more productive pool of researchers is sooner or later overtaken by the reduction of the market size effect. Conversely, getting further away from an extreme polarization of human capital in the hands (or heads) of a few is growth-enhancing because the expanded demand for new capital goods more than compensate the lower inventors's creativity. Fig(1.C), which plots g_N against ε, shows the inverted-U relationship between the innovation rate and inequality just described, [10] and Fig. (2) illustrates how the shape of the curve is affected by the elasticity ϕ: the greater this is, the wider the ascending section of the curve. In sum, this simple experiment shows that if the inventors' creativity (ϕ) is different across countries we should expect also a different trade-off between inequality and growth: Countries with poor research facilities, or more generally closed to new ideas, are more likely to have a low ϕ, and therefore are more likely to be in the declining section of the curve.

6.3 Market Size, Productivity, Mark-up, Externalities and Inequality

The previous experiment showed the negative role of inequality on the innovation activities via the market size. In this section, I explore more in details the connection between inequality, market size and innovation when the distribution of human capital is represented by a Gamma density function of parameters (a, b). One important aspect of the model that will come to light is that the association between the market size and the innovation activity is not necessarily positive.

Fig. (3) illustrates Lemma 1: All individuals on the right of $\bar{h}$ choose to be inventors and the rest choose to be final good producers. Inequality is generated by altering the parameters (a, b) so as to keep the mean $(a \times b)$ constant. More dispersion yields a longer right-tail of the distribution that contributes to increase the researchers's skills. The continuous descending line of the top-left plot in Fig. (4), representing the share of inventors, reflects the rightward movement of the equilibrium condition $\bar{h}$ as the human capital variance increases. Conversely the (continuous) ascending line plots the inventors' average quality relative to that of final good producers. The inventors' average skills improve with the variance for two reasons: Each inventor is more able; the pool of inventors becomes smaller and smaller and the ones who abandon the business are the least able. As in the previous example the adverse effect of inequality on the market size can be grasped by the negative relationship between the price of an invention and the Gini ratio – middle of the three left graphs. The bottom graph confirms the ambiguity of the relationship between innovation rate and inequality that emerged with the binomial distribution, except that here is clearer how it is linked to the quality and the share of innovators.

Technological Shocks. Imagine that the final good sector is hit by a technological shock

[10] By taking the partial derivative of the right-hand side of Eq. (24) with respect to ϵ and checking the sign, it can be verified that when ϕ is larger (smaller) than $(1 + \frac{[(\gamma)^{\frac{\alpha}{1-\beta}-1}-1]}{[1+(\gamma)^{\frac{\alpha}{1-\beta}}]})\frac{\alpha}{1-\beta}$, where $\gamma \equiv \frac{1-\epsilon}{1+\epsilon}$, the research-effect dominates (is dominated by) the market-size effect.

that reduces the productivity of human capital α. This has two kinds of consequences. The demand for capital goods drops, causing a decline of the market value of new designs. One would then expect that inventors quit their creative activities. Quite the opposite actually occurs because the expected gain of being in the final-good sector is also reduced. As a result, individuals whose human capital is just below $\bar{h}$ switch into the R&D sector, leading to an increase in the share of researchers and in the innovation rate. The consequences of a negative shock on α are illustrated in the same three graphs just discussed. The dashed lines are obtained with the same parameters used for the continuous lines (see Table (3)) except that α is reduced from 0.3 to 0.25. As expected the migration of individuals into the R&D sector causes a reduction of the quality ratio in a large range of the Gini ratio because – the average quality of researcher declines and that of final good producers increases. In section (8) I will discuss historical and cross-country evidence that tend to support the idea that a negative productivity shock may lead to an increase in the number of researchers, whereas inequality has the opposite effect. The innovation-inequality curve appear more hamper-shaped when α is low. In a low-α economy the market-size effect is weaker relative to the productivity effect –hence the steeper ascending section of the curve. But when the market effect takes over the productivity effect, the more pronounced concavity of the production function implies that a rise of inequality causes a larger loss of potential market for new capital goods. As a result, the declining section of the curve is also steeper.

A reduction of β also triggers two types of reactions. Capital good producers increase the mark-up, as there is a lower level of substitutability across capital goods. But final good producers, for a given price, demand less of each good, due to the diminished productivity of capital. The total effect on the profit function and therefore on the price of an invention is ambiguous. However some algebra applied to Eq. (6) reveals that *any* deviation from a starting value of 0.3 *always* causes a drop in the invention's value. The middle column of Fig. (4) compare the scenarios with a high and a low β.

Externalities. Romer's original model considers only the positive dynamic externalities arising in the R&D sector, namely that past inventions create useful knowledge for the creation of new designs. But recent analysis has indicated that the largest externalities are observed in the phase of the inventions' utilization and that countries differ greatly on how the net social value of an invention is split between inventors and individuals other than inventors. How does the rule of appropriation affect the growth-inequality relationship? To answer this question I compare two scenarios with different T. A shorter protection is equivalent at saying that amount of innovation spillovers captured by individuals other than inventors is larger. Formally, the spillovers are measured by c, the fraction of capital goods rented at a marginal cost. Although a reduction in T favors an expansion of the demand of formerly-protected products, the expected stream of profits generated by new products shrinks and so does the inventors' reward. As a result inventors abandon the R&D sector. The three graphs in the third column of Fig. (4) confirms this intuition – The dashed lines, which represent the case with a lower T , lie always below the continuous lines. Notice also that the ascending section of the growth-inequality curve becomes steeper with a lower T because the market-size effect is weaker relative to the research-effect. I should also emphasize that the graphical analysis has abstracted completely from transitional considerations. Clearly output would expands briskly in the short run if the length of protection were shortened. (With the parameters used in the simulation there would be a seven-fold increase in output!)

6.4 Comment: Centralization and Decentralization

It was assumed that both the production of ideas and that of final output is completely decentralized and that the individual running the firm is a residual claimant. This is a departure from the more common set up in which final output is produced by one firm that hires the overall stock of physical and human capital. In a world with homogenous individuals, no financial constraints, constant returns to scale on private inputs, and where output is sold in a perfectly competitive market, centralized production is a quite convenient assumption –although clearly not realistic and somehow in contradiction with the assumption of perfect competition – for it implies the same aggregate relationships that would be obtained by assuming an unspecified large number of producers. But an approach with centralized production would not allow us to uncover macro relationships deriving from the unequal distribution of human capital, because the production decisions are based only on average human capital. To see this point more formally, consider the production function

$$Y = zH_Y^{\alpha} \int_0^N X(i)^{1-\alpha} di, \tag{25}$$

where H_Y is some aggregate level of human capital and $X(i)$ denotes the quantity of intermediate input i. It is easy to verify that the demand for good i is $(\frac{z\beta}{p(i)})^{1/(1-\beta)}(H_Y)^{\alpha/(1-\beta)}$. The extent to which alterations of the distribution $F(.)$ affects H_Y depends crucially on how the aggregation of human capital is carried out. If human capital is added linearly, namely if $H_Y = \int_0^{\bar{h}} hdF(h)$, the concavity at the individual level is transformed into an aggregate concavity, and the link between human capital inequality and market size washes out almost entirely (there may still be some second-order effects through movements of $\bar{h}$). In a sense the centralized firm can be seen as bank that collects all the human resources and uses it in the most efficient way. In sum, if production were centralized there would *always* be a positive relationship between inequality and growth, for there is no mechanism in the economy that counterbalances the research-effect. But – in line with the conjecture of this work – in market economies production is decentralized.

7 Calibration

So far, I have argued that although higher human capital inequality reduces the demand for capital goods it may nevertheless be desirable, at least up to a point, because it enhances the productivity of inventors. The two simulations discussed above indicated that the ambiguities in the relationship depends on the functional form of the production of innovation and of that of final goods. Under what circumstances is the market-size effect more likely to dominate the R&D effect? I try to provide an answer using two approaches. In this section I calibrate the model using post-war period data on the U.S. economy using different guesses on unobservable parameters. In the next section, by the means of regression analysis, I discuss cross-country empirical evidence on inequality and on the market for information and communication technologies.

7.1 Parameters

The choice of the baseline values for the vector of parameters $(z, \eta, \rho, \sigma, \beta, \alpha, \phi, \delta)$ is quite straightforward. The productivity parameters associated with the production of final output and capital, z and η, respectively, do not play any role in the equilibrium condition (11); therefore both of them are equal to 1. I set the preferences parameters $\rho = 0.02$ and $\sigma = 2$, in line with many other studies. The output elasticity to the capital goods is rarely considered below 0.25 and it is often set at around 0.3. There are no ready estimates for ϕ, α, and δ. In order to pick up a set of reasonable parameters, I require that the model delivers macroeconomic results that are similar to observable macro variables. The requirements are as follows.

a) The average annual growth rate of per capita gross domestic should be around 2% – the long run average observed in the US.

b) The interest rate should be between 3 and 7 percent. The lower bound is around the real returns of the 10-year US Treasury security and the upper bound is the average real stock return estimated by Prescott and Mehra (1985) – presumably part of it is compensation for risk.

7.1.1 Distribution

Perhaps the most challenging aspect of the experiment is the choice of an appropriate human capital distribution. Jones and Schneider (2006) argue that IQ tests provide a useful measure for human capital, and should be used in growth regressions. Psychologists for a long time have been busy in determining the IQ distributions for different segments of society, and in comparing IQs across countries. One of the most comprehensive survey studies of this nature was done by Lynn and Vanhanen (2002) who created an IQ data set for 81 countries.

However, there is a lot of disagreement on what the 'production function' of human capital looks like (Juhn, Murphy, and Piere (1993), Carneiro and Heckman (2004)). For the sake of the experiment, instead of taking one specific view on how to measure human capital, I assume that it is distributed following a Gamma pattern of parameters (a,b), as in one used in the simulation of the previous section. The idea is to control the shape of the Gamma distribution by allowing its two parameters' values to change over a wide range of values and then identifying a region for the two parameters that, in combination with a set of values for the other parameters of the model, delivers an income distribution similar to the one observed for the US. The last column of Table (4) reports the Gini index for household income computed for the Unites States between 1967 and 2002. Voitchovsky (2005) warns us that focusing on a summary inequality statistic may limit our understanding of the relationship between growth and inequality. Therefore I also ask the model to reproduce the income distribution by quintiles (these are recorded in the remaining part of the table).

The advantage of this procedure is that I can match the model to non-controversial numerical information – the income distribution. Clearly there are other elements not considered in the model, such as taxes or trade, that may alter the link between the distribution of income and that of human capital. Nevertheless such elements could be added. In the concluding section I will discuss how the outcome may differ in an open economy.

7.2 Results

To gain room of maneuver in changing the shape of the Gamma distribution I set its mean equal to a relatively high value (100) and allow b (and of course a) to vary in a very wide interval. Given α and ϕ (whose choice will be discussed below), the parameters discussed above, and the interest rate, the growth rate g_N is pinned down by δ. The constraint (A1) requires that $\phi > \alpha/(1-\beta)$. Yet, there is quite a lot of freedom in picking up α and ϕ. Therefore I propose two sets of simulations: one with a large α and the other with a small α. The left-plots of the Fig. (5) compare the outcome of two calibrations, by showing how the number of researchers, their quality, and the innovation rate vary with the Gini ratio. In one – represented by the continuous lines – α was set to 0.3, and ϕ and δ were picked so as to get close to a growth rate of 2%, an interest rate close to the 3-7 percent rage, an inequality index between 0.4 and 0.46, and a distribution of income close to the one reported in Table (4). In the second calibration – represented by the dashed lines – α and ϕ were set to 0.6 and 1, respectively. The top-left graph reveals that there is no significant difference in the way people sort themselves between the two sectors. However, the middle-graph shows that the quality curve is steeper in the economy with a lower α. Indeed, in this economy to obtain a given change in the Gini-ratio requires a larger alteration of the human capital variance. As a consequence the conditional mean of human capital for individuals above $\bar{h}$

is greater. This also explains also the difference in slopes of the two plots representing the growth-inequality relationship contained in the bottom-left graph. In the low-α economy a 0.1 change in the Gini coefficient in the range 0.25-0.5 – many advanced economies fall into this range– is associated with about a quarter percentage faster innovation rate, whereas in the high-α economy the acceleration is about 0.13%. To discern between the two cases I compared snapshots of the income U.S. distribution since the late 1960s with the outcome of the two calibrations. The two graphs on the right column of the Fig. (5), which plot the quintiles of the distribution against the Gini ratio, shows that the model does a better job in replicating the income distribution when α is 0.3.

Conclusion 1 *Under a parametrization of the production functions that replicates well the post-war growth average income and its distribution the slope of the inequality-innovation rate curve is about 0.025, implying that a change in the Gini coefficient of 0.1 is associated with a 0.25% change of the innovation rate. This is about half the slope's value estimated by Barro (2000) for the group of medium and high income countries. When α and ϕ are relatively large (*0.6 *and* 1*, respectively) the acceleration implied by the calibration is lower (approximately 0.13%) and the fitting of the income distribution is poorer.*

8 Empirical Evidence

It was conjectured that the negative consequences of inequality on growth are due to the diminished capacity of the economy in absorbing new capital goods. I test this proposition by using US and international data on the consumption of electronic products and on income inequality. Data on U.S. inequality by state is drawn form the U.S. census, which calculates the Gini ratios on the basis of household surveys conducted at a 10-year interval. The Current Population Survey (CPS) records data on computer ownership at the household level since 1984, but the oldest available data by states was recorded in the September 2001 survey. [11] The cross-section data are taken from various issues of the Human Development Report (HDR) produced by the United Nations Development Program (UNDP).

United States. The outcome of a least square regression in which the dependent variable is computer ownership and the dependent variables are average income and the Gini ratios are reported in Table (5). Although in 2001 computers were already widely spread[12] there is enough variation in the data to deliver coefficients associated with the two explanatory variables that are separately and jointly significantly different than zero. Furthermore the

[11] One place where they can be found is http://www.census.gov/hhes/www/income/histinc/state/state4.html

[12] The data can be found at http://www.census.gov/population/www/socdemo/computer.html

Correspondence with a Census official confirmed that state level data was very spotty until 2001 and therefore have not been included in the official statistics.

inequality coefficient has the sign predicted by the model: For a given level of income the market for computers is larger in states with a lower income inequality.

Cross-Country Estimation. The UNPD does not survey the households ownership of computers but only internet use. Assuming a high correlation between the two sets of survey data, I ran a similar regression to the one just described on a group of 95 countries. The top-right panel of Table (6) indicates that the slope associated to inequality is negative although only marginally significantly different than zero (P-value is about 0.15). Similarly, I obtain a negative coefficient, but not statistically significant, when the dependent variable is the percentage of cellular phones. But the bottom two panels, that reports estimation results when mainline telephones in 1990 and 2004 are proxies for market size, indicate a much stronger relationship: the inequality coefficient has the predicted negative sign and is highly significant (P-value$<$0.01).

A second class of implications concerns the link between innovation activity and inequality. It was stated that an equal economy is likely to have more inventors than an unequal one because the demand for capital goods there is greater. I verify the hypothesis by regressing the number of researchers on inequality and average income using UNDP data on 68 countries. The outcome of this regression, which is collected in the left-hand side of Table (7), indicates that the hypothesis is not rejected with a high level of confidence (the P-value below 0.01).

In section (6.3) was observed that the model does not always imply a rise (fall) in the number of researchers when the demand expands (shrinks). If the fall in demand is caused by a negative shock on human capital productivity in the final good sector, the business of invention becomes suddenly more attractive, notwithstanding the lower value of an innovation. There is some appealing historical evidence suggesting that a phenomenon of this kind has occurred in the United States. The economic historian Fano (1987) documents a dramatic rise in research personnel during the Great Depression years – of a magnitude of about 300% between 1921 and 1938 – and in the number of laboratories – these rose from fewer than 300 in 1920 to more than 2,200 in 1938. The boom in industrial research was not entirely driven by product innovation – the type of innovation assumed here – for many entrepreneurs seemed to be especially interested in cost-reducing innovations. However, Fano argues that in this period a large number of new products were being developed as well, especially in the large-scale enterprises, such as electric refrigerators, rayon, washing machines, and radios.

It is considerably more difficult to find cross-country data on the economic value of an innovation. The monetary value of patents has been estimated with indirect measures, such as the number of citations that patents receive after their publication (see Hall, Jaffe, and Trajtenberg (2005)), the renewal fees paid by the patent holders to extend the patent protection (Pakes (1986)), the number of backward citations to other patents, and through a composite indicator of quality of patents (Lanjouw and Schankerman (2004)). Building on these contributions one could test the hypothesis that a country with an unequal distribution of human capital generates fewer patents (per person) than those generated in a relatively more equal country, and that the patents produced in the unequal countries are nevertheless cited more often.

9 Conclusion

This study proposed to interpret the conflicting evidence on the relationship between inequality and growth documented by the empirical macro literature as the result of two contrasting forces that are put into motion by a rise in human capital dispersion. One is the improved productivity of inventors, and the other the diminished market value of a design. It was shown that an inverted-U relationship between the innovation rate and inequality is likely to emerge, that is, going away from an extreme form of equality or inequality is growth-enhancing. A departure from an extreme form of inequality induces a sizable increase in the demand for new capital goods, and in the market value of patents. Therefore a larger number of talented individuals are willing to devote themselves into creative activities. Conversely, a society that is extremely worried in reaching equality in knowledge acquisition is unlikely to reap the benefits of extraordinary minds.

Through a number of simulations I have shown how the intensity of innovation activities and the shape of the inequality-growth change when some fundamental parameters of the economy are altered. Not surprisingly, if the inventors' ability to appropriate the social value of a design weakens, the market-size effect is weaker. A less obvious result emerged while investigating the way the economy responds to a drop in the demand for capital goods. If this is caused by inequality, the overall market for capital goods suffers, because on *average* producers are less productive and choose (on average) a smaller production scale. But when drop is due to a negative technological shock on human capital productivity the creative activity intensifies because the reduced prospect of earnings as producers induce some former producers to become inventors. A combination of historical and empirical evidence indicated that either mechanism may be in place at any given time. Hence an open question to be investigated more carefully is under which conditions a decline in the demand for capital goods is detrimental for innovation.

One simulation and a calibration exercise provided suggestions on how to identify the inequality-innovation curve. A country with efficient research institutions and open to the acceptance of new ideas is more likely to be on the ascending section of the curve. The calibration exercise based on post-war U.S. data indicated the possibility that in recent decades the U.S. (and countries with similar macro parameters) has been in that portion of the curve. In particular, the slope calculated under a preferred set of parameters ($\alpha = \beta = 0.3$ and $\phi = 0.6$) implied that a 0.1 rise of the income Gini index – a variation that is greater than the one observed in the U.S. since the 1960s – is paralleled with a faster innovation rate of about 0.25%. This is about half the magnitude of the partial correlation estimated by Barro (2000) between the (residual) average rate of per capita income and the Gini coefficient for a set of medium- and high-income countries, and about a fifth of a similar estimation performed by Forbes (2000).

Finally, I would like to mention two additional open issues for future research. This work could be extended to investigate how the shape of the skill distribution affects the direction of trade of capital goods and of inventions. In a working paper version of this study, I conjectured that the more unequal economy tends to specialize in the business of creation and the equal economy in that of actual production. Another aspect worth investigating is how the dynamic consequences of a change in the distribution depend on nature of the social or technological developments that altered the distribution. How does the market-size effect relate to the creativity effect when the shape of the distribution is disturbed by social innovations such as the elimination or the introduction of some form of religious, gender or ethnic discrimination?

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	Mean		Change	SD		Change
	Grade 4	Grade 7	(%)	Grade 4	Grade 7	(%)
African American	196.50	214.30	9.06	12.51	15.89	27.02
Hispanic	196.50	214.60	9.21	12.03	15.55	29.26
Native American	195.90	213.40	8.93	11.74	14.98	27.60
Non-Minority	204.10	226.00	10.73	11.69	15.26	30.54

Source: Author's elaboration based on Table 15 in McCall et al., 2006, p17.

Notes: The scores (in RIT) scale refer to math tests taken in the fall of 2003.

Table 1: Mean and Standard Deviation in Math Test Scores

Country	Mean score	Variance
Japan	534.14	116.31
Canada	532.49	88.75
Spain	485.11	90.80
United States	482.89	104.93

Source: OECD (2005) Table A6.1; OECD PISA (2003), Table 4.1a.

Notes: The variance is a percentage of the OECD average.

Table 2: Dispersion in Educational Achievement

α	β	η	z	T	δ	ϕ	$a \times b$	r
0.3	0.3	1	1	20	0.02	0.6	100	0.05

Table 3: Baseline Parameters

Year	Lowest	Second	Third	Fourth	Highest	Gini
2002	3.5	8.8	14.8	23.3	49.7	0.462
1990	3.9	9.6	15.9	24	46.6	0.428
1980	4.3	10.3	16.9	24.9	43.7	0.403
1970	4.1	10.8	17.4	24.5	43.3	0.396
1967	4	10.8	17.3	24.2	43.8	0.399

Source: Table A-3, Carmen DeNavas-Walt et al. (2003)

Table 4: Quintiles and Gini Coefficient

	Computers Owned by Family		
Variable	Coefficient	Std. Error	t-Statistic
Constant	59.9532	9.3032	6.4444
Income	0.3593	0.0542	6.6264
Gini	-0.7286	0.1846	-3.9462
R-Square	0.5993		
F-Stat	35.9008		
Obs.	50		

Table 5: Market Size and Inequality in the U.S.

Note: The coefficients are estimated with a least square regression. Dependent variable: percentage of families owning a computer. Indipendent variables: average family income, as a percentage of that of the state with the highest income; the Gini ratio expressed in a 0 -100 scale.

Cellular Phones 2004			
Variable	Coefficient	Std. Error	t-Statistic
Constant	18.0335	9.9611	1.8104
GDP	1.9417	0.1344	14.4435
Gini	-0.1617	0.2240	-0.7217
R-squared	0.7485		
F-statistic	127.9936		
Obs	89		

Internet Use 2003			
Variable	Coefficient	Std. Error	t-Statistic
Constant	8.8575	5.6021	1.5811
GDP	1.1694	0.0766	15.2738
Gini	-0.1852	0.1285	-1.4413
R-squared	0.7485		
F-statistic	127.9936		
Obs	95		

Telephone Mainline 2004			
Variable	Coefficient	Std. Error	t-Statistic
Constant	15.6767	4.3071	3.6398
GDP	1.0997	0.0569	19.3268
Gini	-0.2753	0.0966	-2.8500
R-squared	0.8682		
F-statistic	253.6773		
Obs	80		

Telephone Mainline 1990			
Variable	Coefficient	Std. Error	t-Statistic
Constant	7.8112	3.3508	2.3312
GDP	0.9857	0.0453	21.7691
Gini	-0.2239	0.0766	-2.9246
R-squared	0.8702		
F-statistic	305.0770		
Obs	94		

Table 6: Market Size and Inequality: Cross-Country

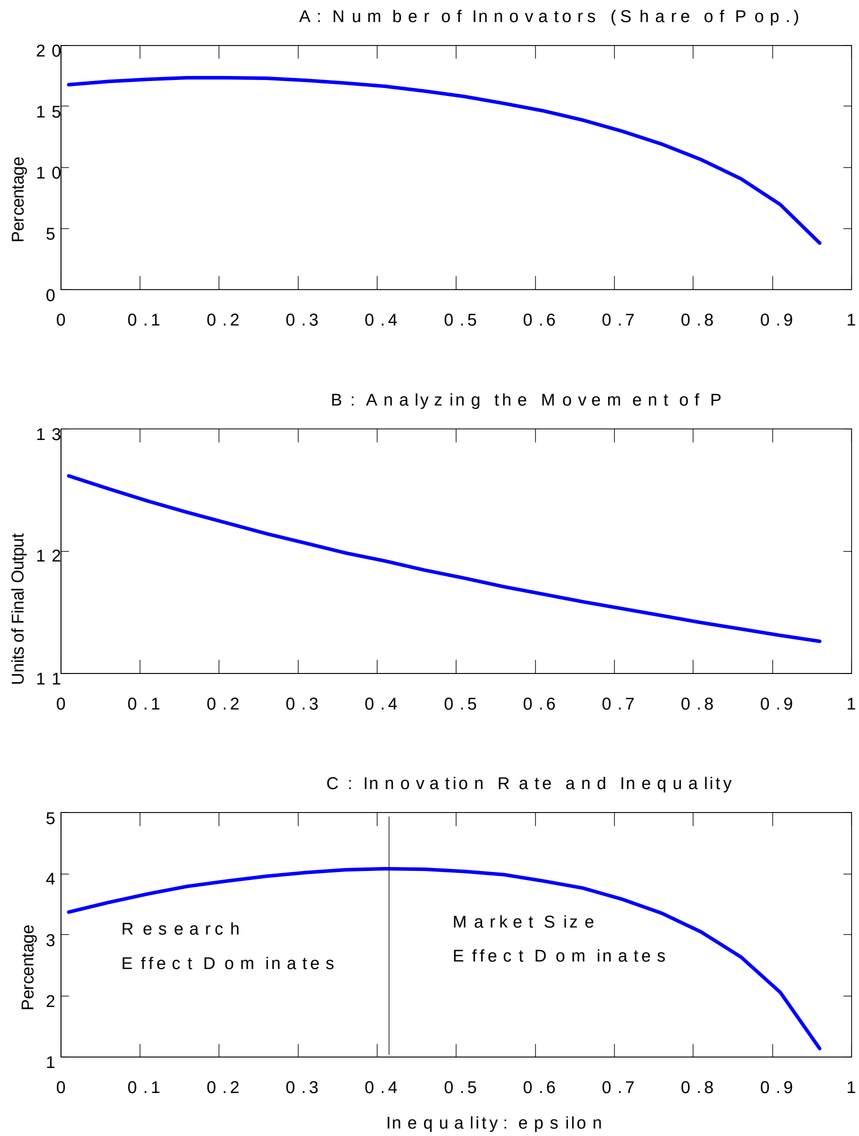
Source: Author's elaboration based on data drawn from the Human Development Report 2006.
Notes: The dependent variables, indicated at the top of each table, is expressed as a percentage of that of the country with the highest figure. The independent variables are i) per capita average gross domestic product (GDP) in purchasing power parity terms, as a percentage of per capita GDP of the richest country (Luxemburg) in 2004; ii) The Gini ratio. This is computed by the UNDP on the basis of household surveys taken at different years, mostly in the last decade.

Number of Researchers			
Variable	Coefficient	Std. Error	t-Statistic
Constant	28.9506	7.9921	3.6224
GDP	0.8179	0.1012	8.0816
Gini	-0.6572	0.1814	-3.6227
R-squared	0.6489		
F-statistic	60.0555		
Obs	68		

Table 7: Inequality and Number of Researchers

Source: See Table (6). Notes: For explanations on the independent variables see Table (6). Researchers are people trained to work in any field of science who are engaged in professional research and development activity. Most such jobs require the completion of tertiary education. The figures on the number of researchers refer to the most recent year available during the period 1990-2003. In the regression they are expressed as a percentage of that of the country with the highest value.

Figure 1: Inequality and Innovation with Two Types of Individuals

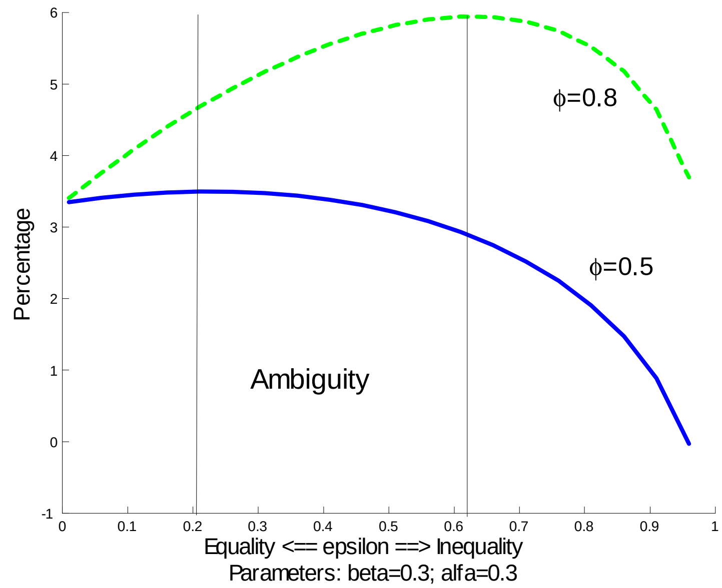


Source: Author's elaboration based on Eqs. (22), (23), and (24).

Parameters' values: $H = 1; \beta = 0.3; \alpha = 0.3; \delta = 0.2; \phi = 0.6; r = 0.05; \eta = z = 1.$

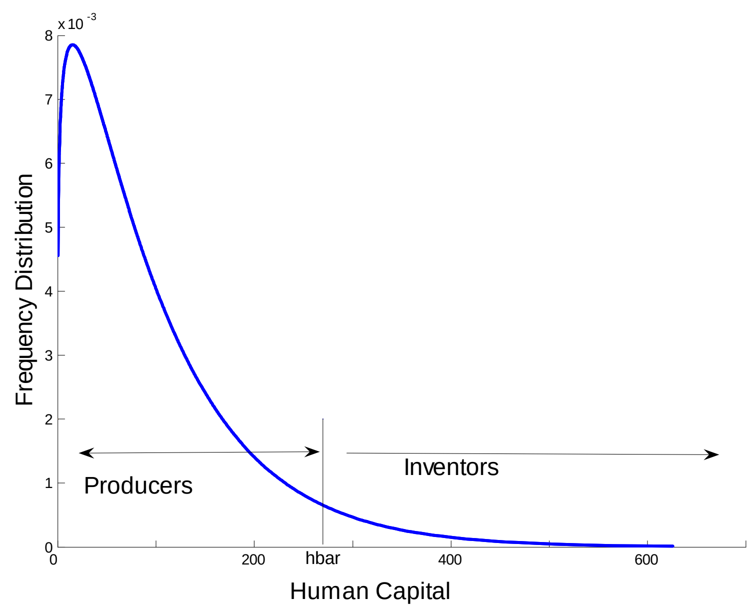
Note: In all three plots the running variable is ϵ, a measure of the deviation from the average value of human capital.

Figure 2: Innovators Productivity, Inequality and Growth



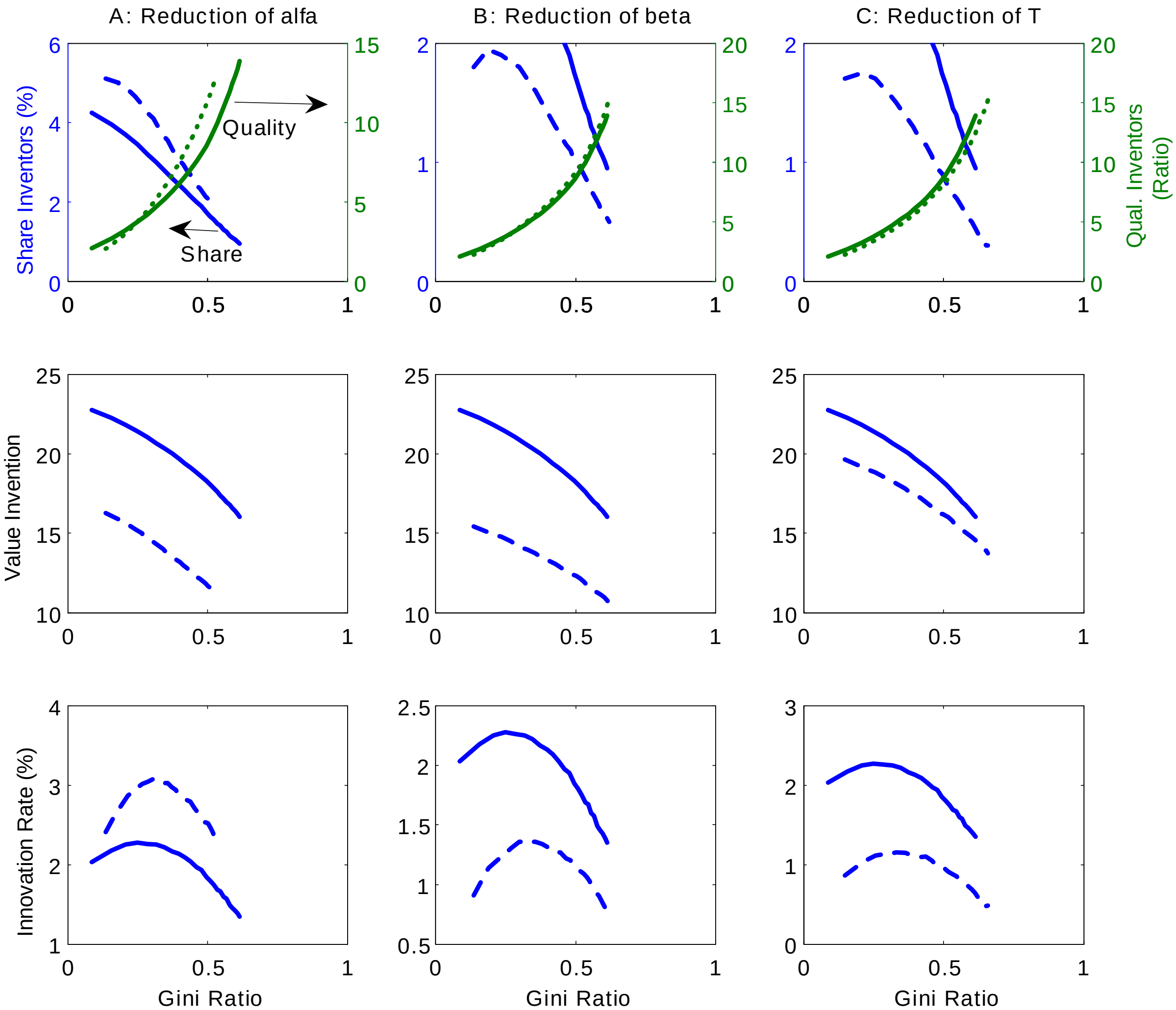
Source: Author's elaboration based on Eqs. (22), (23), and (24).

Figure 3: Allocation of Individuals Between Sectors



Note: Gamma distribution of parameters a=1.1765 and b=85

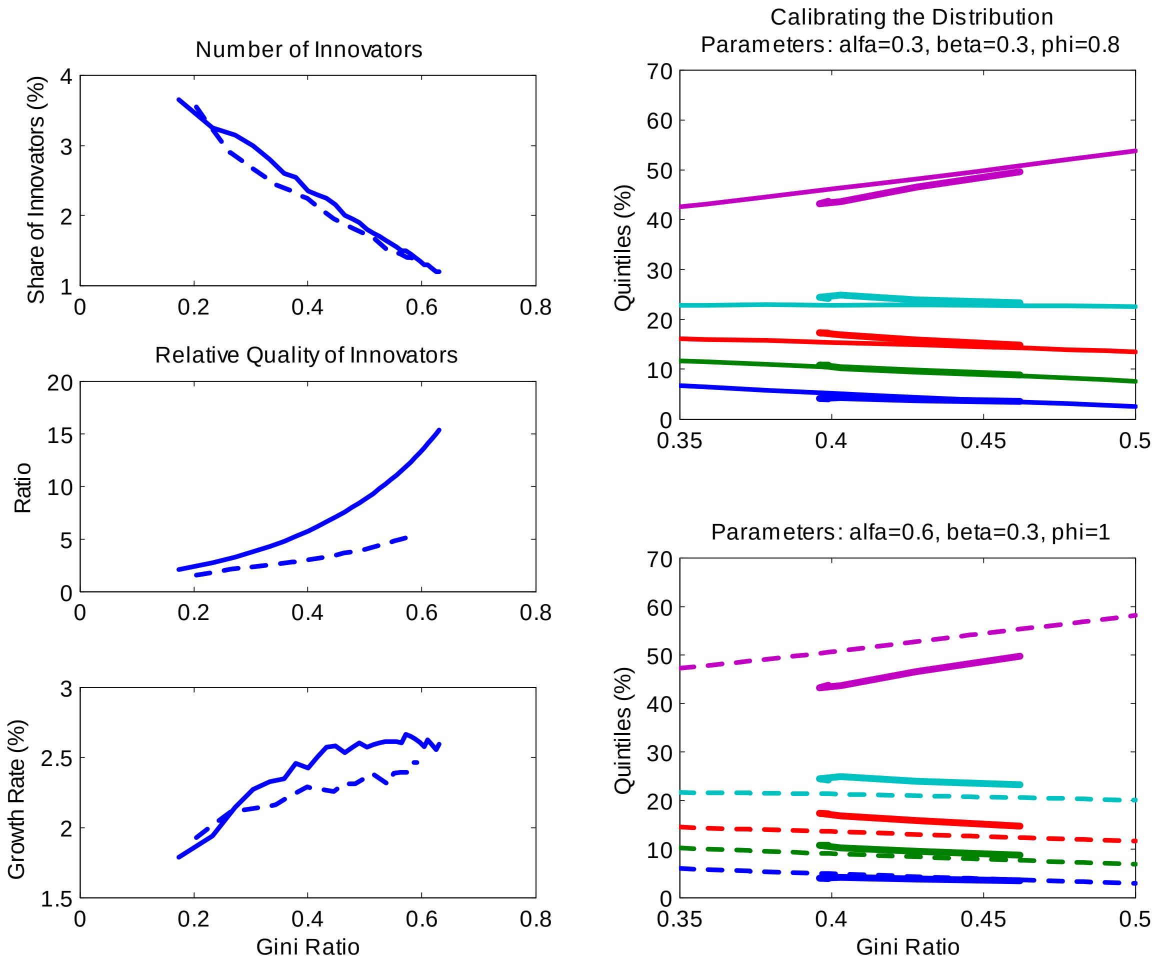
Figure 4: Simulations with a Gamma Distribution



Source: Author's elaboration based on the model presented in the text.

Note: Each of the three graphs in the top row plots the number of inventors as share of the population and the average human capital of inventors relative to that of final good producers (ascending lines). The three graphs in the middle row show the behavior of the price of an invention. The three graphs at the bottom row show the movements of the innovation rate. All quantities are plotted against the Gini index for income. The continuous lines refer to the base line case that assumes the parameters' values reported in Table (3). The dashed lines present scenarios in which the baseline parameters are replaced (one at a time) as follows: $\alpha = 0.25$, $\beta = 0.25$, $T = 15$.

Figure 5: Calibrating the Concavity of Production Functions



Source: Author's elaboration based on the model and Table (4).

Note: The dashed (continuous) lines in all graphs refer to the case with high (low) α andϕ. The thicker curves appearing the graphs on the right column are based on the data reported in Table (4).