# A HUNDRED YEARS OF BUSINESS CYCLES AND THE PHILLIPS CURVE

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**SCIENCES PO OFCE WORKING PAPER n° 17/2024**

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WORKING PAPER CITATION

This Working Paper:
Lapo Bini, Lucrezia Reichlin and Giovanni Ricco,
A Hundred Years of Business Cycles and the Phillips Curve
*Sciences Po OFCE Working Paper*, n° 17/2024.

DOI - ISSN

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## ABSTRACT

This study investigates the business cycle dynamics of the U.S. economy since 1900 through a multivariate framework that imposes minimal economic restrictions. A key finding is the presence of a significant negative correlation between inflation and economic slack, at business cycle frequencies. This relationship remains robust across over a century of data, with stable coefficients in subsample periods.

## KEYWORDS

Phillips Curve, Semi-structural models, Business cycle, Okun's law

## JEL

E31, E32.

# A Hundred Years of Business Cycles and the Phillips Curve

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November 21, 2024

**Abstract**

This study investigates the business cycle dynamics of the U.S. economy since 1900 through a multivariate framework that imposes minimal economic restrictions. A key finding is the presence of a significant negative correlation between inflation and economic slack, at business cycle frequencies. This relationship remains robust across over a century of data, with stable coefficients in subsample periods.

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We thank Olivier Blanchard, Mark Gertler, Thomas Hasenzagl and Yannick Kalantzis for providing helpful suggestions. We also thank participants to the VII Annual Santiago Macro Workshop and the seminar series of the University of Porto for useful comments.

*Business cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle. This sequence of changes is recurrent but not periodic. In duration, business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own. (Burns and Mitchell, 1946)*

# 1 Introduction

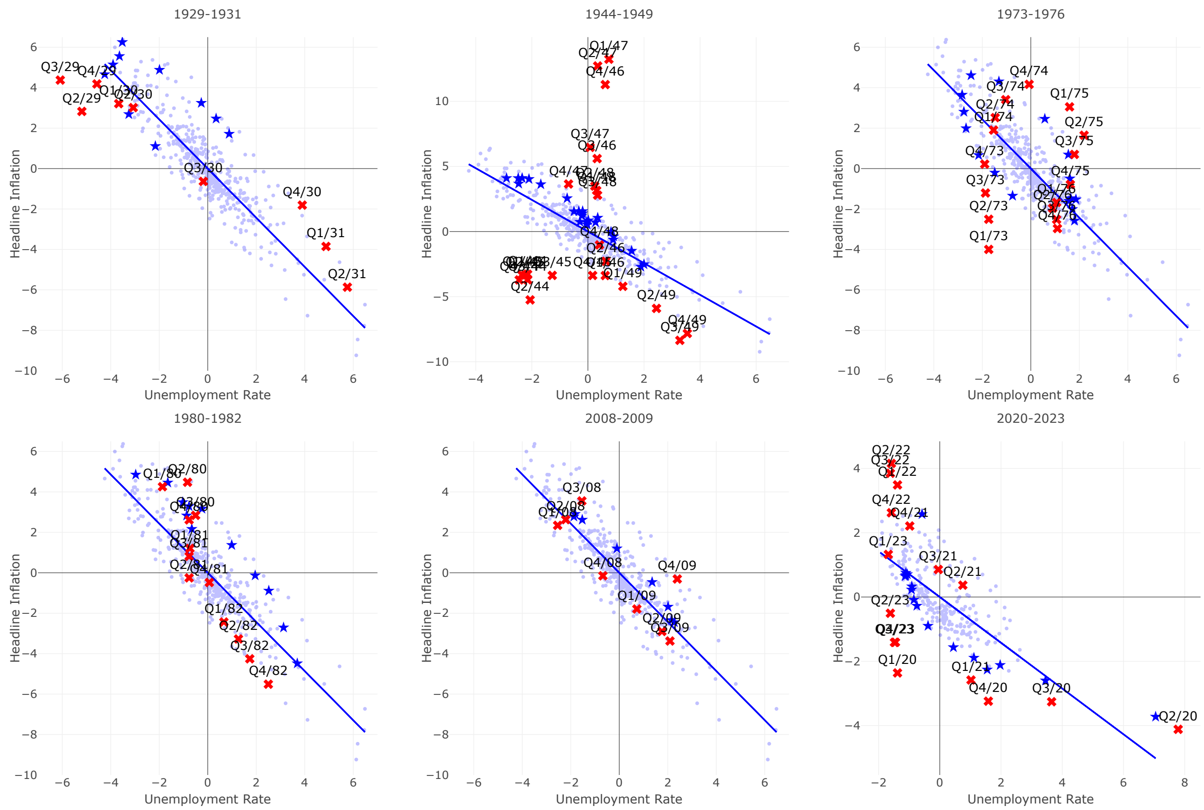
Almost 70 years ago, in a classic paper analysing UK data, Phillips (1958) published evidence of a negative correlation between a measure of the slack in the economy and a measure of inflation. This correlation was confirmed for other economies in early literature through various simple reduced-form regressions, where slack in the economy was proxied by unemployment or the output gap, and either price or wage inflation were considered as dependent variables.

Later academic research focused on the rationalisation of this correlation as resulting from the optimising behaviour of economic agents, in the presence of price frictions. Since the 1990s, a microfunded Phillips curve equation has become a key building block of the New Keynesian model.[1] Paradoxically, as the theoretical foundations consolidated, the empirical relationship disappeared from the data. The Phillips curve has been declared alive by some and dead by others, killed by policy or by luck, steep or flat, and, more recently following the post-pandemic inflation surge, non-linear or unstable. Over the last fifty years, papers on this subject can be counted in the thousands.[2]

[1] The New Keynesian Phillips curve (NKPC) equation connects inflation to inflation expectations and the gap between the frictionless optimal price level and the current price level.

[2] We do not have the ambition to review this very rich literature here. For evidence on the NKPC, we refer the reader to the excellent recent reviews by Mavroeidis et al. (2014), and Furlanetto and Lepetit (2024) and for evidence on Phillips curve based forecasting to Stock and Watson (2009).

Figure 1: Selected episodes of high inflation and recessions in the US.




**Notes:** The chart plots demeaned CPI inflation against the demeaned unemployment rate (red cross), the reduced-form Phillips curve (blue line) obtained from model-based business cycle components of unemployment and CPI inflation (blue dots). The first five charts are obtained from a model estimated over the sample 1900-2019. The last one is obtained from over the sample 1960-2023 (the parameters of the model are estimated over the sample 1960-2019).

Empirical research has failed to provide robust evidence on the existence of the Phillips curve. On the one hand, the forecasting literature has generally reported the poor out-of-sample performance of models based on some form of reduced-form Phillips curve. In a forecasting evaluation including many models, Stock and Watson (2009) concluded that inflation forecasts based on the traditional specification of Gordon (1990) do well in some sub-samples but generally do not improve on univariate models that include only inflation. On the other hand, the macroeconomic literature, focusing on the identification of the parameters of the New Keynesian structural equation, found inconclusive results due to weak identification. As Mavroeidis et al. (2014) concluded, "the literature has reached a limit on how much can be learned about the New Keynesian Phillips curve from aggregate macroeconomic time series.

New identification approaches and new datasets are needed to reach an empirical consensus."

A cursory inspection of some episodes of higher inflation and deep recessions in recent US history illustrates the problem (Figure 1). The red stars indicate the values of demeaned inflation and unemployment for each selected period. While the recessions of 1929-31, 1980-82, and 2008-09 feature a standard reduced-form Phillips curve with a steep negative correlation between inflation and unemployment, the correlation disappears, and the Phillips curve becomes vertical in the high-inflation episodes of 1944-49, 1973-76, and 2020-23.

The narrative of these episodes points to diverse combinations of events and possibly distinct constellations of demand and supply disturbances. The Great Depression, the Volcker Recession and the Global Financial Crisis, which feature a sharp increase in unemployment coupled with a decline in inflation (in the case of the Great Depression inflation reached its trough in 1931 at $-9.3\%$), were associated with a monetary contraction and/or financial disruptions. In contrast, the role of supply disturbances were likely to be larger for the inflation episodes of post-World War II inflation, 1973-76 and post-COVID. In the post-World War II years we had a combination of the elimination of price controls, supply shortages, and pent-up demand with the last two factors also characterising US post-COVID inflation. Conversely, high inflation in the 1973-76 period is often cited as a textbook case of stagflation caused by the large oil shocks associated with the Yom Kippur war and the Arab oil embargo which began in October 1973.

Against the backdrop of this evidence, this work attempts to assess whether a simple empirical linear model can identify a stable reduced-form Phillips curve over a long sample, once trends and idiosyncratic components are accounted for. The scatter plots of blue dots and stars in Figure 1 visually summarises the main result of this approach. They show the values of the estimated cyclical components of unemployment and inflation for the full sample (blue dots) and for each specific episodes (blue stars). The charts reveal a steep and stable relationship between slack in the economy and price pressure.

The model identifies the unobserved common cyclical components and non-stationary trends via: (i) multivariate restrictions informed by a stylised model of the economy, (ii) empirical measures of expectations, and (iii) assumptions about the orthogonality of the different unobserved components. Our methodology follows Hasenzagl et al. (2022) by adopting a medium-scale multivariate time series model in the tradition of Harvey (1985). In particular, the model identifies a common business cycle that can be seen as a model-based measure of the output gap and its reverberation to prices and price expectations via a reduced-form Phillips curve, to the labor market via Okun's law, and to the short-term interest rate via the systematic component of monetary policy. Furthermore, the model estimates non-stationary trend components that can be interpreted as the output potential, the natural rate of unemployment (or NAIRU), and trend inflation. We view this approach as a tool to identify stylised facts from macroeconomic data in the tradition of Burns and Mitchell (1946), Phillips (1958), and the business cycle literature that has adopted different filtering procedures to separate trends from cycles and studied the cyclical properties of economic variables (see, for example, Stock and Watson, 1999 and Canova, 1998).

In adopting a bare-bones modelling approach and quarterly time series spanning a long period of US history, starting in 1900, our analysis focuses on the stability of key relationships among macroeconomic time series. Rather than focusing on what changes, we aim to identify what does not and what is robust over large spans of time.

We report five main results. First, the model identifies a stable and sizeable common cycle for the whole sample and the sub-samples we consider. The estimate of the output gap, which provides a direct measure of this cycle, perfectly matches the NBER dates of recessions. It also aligns for most of the sample with the available official estimates of the gap, and hence with the frequencies and variance commonly assumed for business cycle fluctuations. Moreover, along the business cycle, the presence of slack in the economy corresponds to increasing unemployment, lower interest rates, and deflationary pressure on prices. This pattern of correlation indicates that the common cycle is generated by a combination of disturbances whose aggregate effects match what would be generally labeled as a 'demand' cycle which the model identifies as the main driving force of US business cycles.

Second, and central to the scope of this study, a given amount of slack in the economy corresponds to a given price pressure. The relationship is robust and stable over the full sample, and especially in the post-World War II period. This is, in essence, strong evidence of the existence of a reduced-form Phillips curve.

Third, the dynamics of inflation is explained only in part by the common cycle at business cycle frequency. It also follows an idiosyncratic component, which turns out to be highly correlated with oil prices in the post-World War II sample. Movements in oil prices act as a 'shifter' around the Phillips curve and can obfuscate the basic correlations at business cycle frequencies. This confirms the intuition of the Gordon's Phillips curve (see Gordon, 1990): when energy prices are highly volatile, the Phillips curve correlation disappears since inflation fluctuates with energy prices and not in line with the common business cycle component.[3] While this 'energy' component in inflation dynamics is in our model is uncorrelated to output, one must be careful not to interpret this as saying that oil shocks do not have real effects since this energy cycle is generated by a convolution of shocks which we do not identify. In fact, the model is likely to attribute at least partially the real effects of oil shocks to the business cycle component.

Fourth, the cyclical component of short-term interest rates comoves with the output gap cycle (and therefore inflation) but is not correlated with the energy cycle both in the full sample and in the sub-samples. We interpret this result as indicating that systematic monetary policy, historically, has not responded to oil price fluctuations which are not correlated with the slack in the economy.

Finally, we find that despite the overall stability of the results, particularly for inflation, during the Great Moderation, the cyclical responses of unemployment and interest rates to the output gap indicate a stronger cyclical variation in the labor market and policy rates to demand conditions. These coincide with an observed lower variance of the business cycle. We

[3] In our analysis by sub-samples, we find that, while the relationship between inflation and the business cycle is stable, that with the energy cycle is unstable.

interpret these results as indicating that both structural changes in the labor market and tighter monetary policy have a role in explaining the low volatility of the Great Moderation. However, our methodology does not identify structural shocks and structural parameters, and hence does not allow for a definitive conclusion on this matter.

The stylised facts that we have uncovered may be coherent with different structural interpretations but they definitely rule out the hypothesis that the Phillips curve was ever dead. They also suggest that the contradictory results found in the literature are explained by not correctly isolating the common dynamics in inflation, output and unemployment from long-run trends, and noise in price variation.

The paper is organised as follows. The remainder of this section provides some references and background to our modelling approach. In Section 2, we provide a stylised representation of the econometric model and its motivation in terms of a toy macroeconomic model. In Section 4 and 3 we describe the econometric specification and the data. In Section 5 and 6 we present key results of the estimation for the long sample 1900-2019 and the post World War II subsample. In Section 7, we analyse the stability of the model over different subsamples throughout the post World War II period, and in Section 8 we present some key results on the COVID and post-COVID sample. Section 9 concludes. The Online Appendix contains the full set of results of all the models and for all the sub-samples considered .

**Modelling approach and related literature.** Our modelling approach follows the tradition of semi-structural models that combine reduced-form statistical methodologies with theory-informed restrictions to identify unobserved economic quantities of interest (see for a review Hasenzagl et al., forthcoming). Examples of this approach include the works in the tradition of Harvey (1985), but also VARs (see Del Negro et al., 2017) and factor models (see, for instance, Barigozzi and Luciani, 2023) with stochastic trends.[4]

[4]Recently, there has been a renewed interest in these techniques in macroeconomics. Relevant references include Morley et al. (2003) and Grant and Chan (2017) (US output gap), Mertens and Nason (2015) and Mertens (2016) (US trend inflation and inflation dynamics), Jarociński and Lenza (2018) (Euro Area output gap), Hasenzagl et al. (2022) (US trend inflation, output gap and the Phillips curve), Ascari and Fosso (2024)

Similar to related approaches, our methodology identifies common cyclical components by separating them from both low-frequency (trends) and high-frequency (idiosyncratic and seasonal components) variations in the data. However, compared to other approaches, our model identifies a minimal number of common components through a minimal set of multivariate restrictions and assumptions, allowing us to clearly single out the business cycle commonalities in the data, which are at the core of this study.

An alternative approach to identifying cyclical correlations in the data, based on VAR analysis, has been proposed by Giannone et al. (2005), Giannone et al. (2006), Giannone et al. (2019), and Angeletos et al. (2020). This approach aims to identify the set of shocks that account for the maximum correlation at business cycle frequencies to then study the conditional comovement of the variables of interest, for example, prices and output. However, as Bianchi et al. (2022) has pointed out, this approach has some limitations: a standard fixed-coefficient VAR may fail to disentangle business-cycle and low-frequency movements over a relatively short period, particularly when structural breaks are present, as it does not account for low-frequency movements in the data. The fact that our findings point to a large and relatively stable business cycle correlation between nominal and real variables reflects the flexibility that the model allows for trends (as in Bianchi et al., 2022) and the discipline imposed by the cross-equation restrictions.

It is worth pointing out that trend-cycle decompositions are not unique by nature. Our modelling assumptions about linearity, the dynamic shape of the components, and orthogonality among them, as well as our Bayesian priors, identify a non-unique approximation to the structure of the data (for an early discussion on the uniqueness of trend-cycle decompositions, see Lippi and Reichlin, 1994). Possible regime shifts, outliers, or nonlinearities are absorbed by the idiosyncratic trends and cycles, which must be understood as wedges between the data-generating process and the statistical model. The model should therefore

and Bianchi et al. (2022) (US Phillips curve), Maffei-Faccioli (2020) (US output potential), Zaman (2021) (US long-run equilibrium levels for rates and other variables), and Bergholt et al. (2023) (US labour market trends and dynamics).

not be interpreted literally but rather as a device to capture the important features of the data and uncover stable relationships if they are present. The extensive robustness analysis we perform helps in assessing the goodness of the approximation of the data-generating process provided by the model against the commonly accepted notions of business cycles.

# 2 A stylised model of trends and cycles

Let us start by presenting a commonly accepted stylised description of the economy in the aggregate, to give the intuition at the core of our empirical specifications. We proceed by first discussing the decomposition of key economic variables into structural trend and cycle components. Then we introduce a stripped down general equilibrium model of the business cycle components of the variables. Finally, we add some cautionary notes about the structural interpretation of the components.

## 2.1 A stylised trend-cycle model

At the very core of the study of business cycle fluctuations there is a decomposition of output into a trend – the output potential –, and a cyclical component – the output gap:

$$y_t = \tau_t^y + \widehat{y}_t^{gap}.$$

The trend, $\tau_t^y$, is usually thought of as determined by technological progress, demographic and institutional factors which inform the long-run behaviour of GDP. It is commonly represented as a unit root process, perhaps with a drift, with permanent innovations:

$$\tau_t^y = \tau_0^y + \tau_{t-1}^y + u_t^{\tau,y}.$$

The output gap, $\widehat{y}_t^{gap}$ – a primitive concept in the description of business cycles –, is instead due to the action of different cyclical factors – demand, supply, monetary, fiscal, energy prices, and many others – pushing output off its long-run equilibrium. This measure of slack, in the Frisch-Slutsky paradigm, is usually considered to be representable as a stochastic but stationary process. For example, one could think of an autoregressive process:

$$\widehat{y}_t^{gap} = \rho(L)\hat{y}_{t-1}^{gap} + v_t.$$

The analysis of business cycles has shown that cyclical fluctuations in many economic indicators – output components, prices, financial and labour market variables – correlate to a different extent, albeit with lags and leads, with the output gap. For example, it is commonly accepted that the output gap is reflected in the cyclical component of unemployment via Okun's law

$$u_t = \tau_t^u + \hat{u}_t^{gap} = \tau_t^u + \delta_u(L)\widehat{y}_t^{gap},$$

while the trend unemployment, $\tau_t^u$ – i.e. the rate consistent with output at its potential –, is called the equilibrium unemployment and is thought to be due to structural and institutional factors in the labour market. It can be described as a unit root process

$$\tau_t^u = \tau_{t-1}^u + u_t^{\tau,u}.$$

Another key macroeconomic relationship, the Phillips curve, connects the output gap to inflation rates in nominal variables

$$\pi_t = \tau_t^\pi + \delta_\pi\hat{\pi}_t = \tau_t^\pi + \delta_\pi\widehat{y}_t^{gap} + \xi_t^{epc},$$

Cost-push shocks, $\xi_t^{epc}$, and different types of supply shocks can move inflation off the relation with the PC curve, creating a negative correlation between prices and output. Trend inflation, $\tau_t^\pi$, is the inflation rate prevailing in the absence of cyclical factors

$$\tau_t^\pi = \tau_{t-1}^\pi + u_t^{\tau,\pi}.$$

It is usually accepted that trend inflation reflects the long-term expectations of agents ($\tau_t^\pi = \lim_{h\to\infty} E_t \pi_{t+h}$), and coincides with the inflation target of a credible central bank.

Interest rates and in particular the policy rate respond to cyclical developments in the economy (and possibly to policy shocks)

$$i_t = \tau_t^i + \delta_i(L)\widehat{y}_t^{gap}.$$

Their equilibrium level is due to both trend inflation and $r^*$, the real neutral rate of interest that equilibrates the economy in the long run.

## 2.2 Demand and mark-up shocks

To understand how such a stylised framework in terms of trends and cycles matches with the standard models we consider a rather general three-equation forward-looking equilibrium model that describes business cycles and detrended variables.

In particular, we consider a simple extension of a standard stylised model of the type discussed in Del Negro et al. (2020) and McLeay and Tenreyro (2020). The model consists of the following three equations for inflation (gap), $\hat{\pi}_t$, output gap $\widehat{y}_t^{gap}$, and the nominal interest rates $i_t$:

$$\begin{aligned}
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \kappa \widehat{y}_t^{gap} + u_t^s, && (1)\\
\widehat{y}_t^{gap} &= \alpha \hat{y}_{t-1}^{gap} + E_t \hat{y}_{t+1}^{gap} - \sigma(\hat{i}_t - E_t \hat{\pi}_{t+1} - u_t^d), && (2)\\
\hat{i}_t &= E_t \hat{\pi}_{t+1} + \theta_d u_t^d + \theta_\pi \hat{\pi}_t^s + u_t^{mp}, && (3)
\end{aligned}$$

where $E_t$ is the operator of mathematical (rational) expectations and $u_t^d$, $u_t^s$, and $u_t^{mp}$ are respectively a demand, a supply or markup, and a monetary policy shock.

Equation (1) is the structural Phillips curve, an aggregate supply relationship that associates positively a measure of 'slack', in the form of output gaps, to inflation. The slope of that relationship is given by the parameter $\kappa$. In New-Keneysian models, it is micro-funded from the firms' optimal pricing problem and links marginal costs to inflation. In that framework, the shock $u_t^s$ originates from fluctuations in desired markups.

The demand equation (2) is the investment-savings (IS) equation of the model, which is derived in the NK framework from the Euler equation of the households. It creates a link between real interest rates and real activity, the strength of which depends on the parameter $\sigma$. The last equation closes the model and captures the response of the monetary policy authority to economic conditions, either as a response to the shocks or directly to the aggregate variable, inflation and output gap. It is generally assumed that the monetary authority responds to inflation with a greater than one coefficient, following the Taylor rule.

In this model, while the demand and the monetary policy shocks cause a positive correlation between slack and inflation, the supply shock can create a negative correlation. The effects of supply shocks, however, depend on the monetary policy response. In the absence of a response of the interest rates to inflation or supply shocks, those would not affect the real variables and would only lead to fluctuations in prices although they may affect trend output.

We illustrate this point, which connects to the question of the optimal response of a Central Bank to demand and supply shocks, by solving the model for the case $\theta_\pi = 0$. We also simplify the model by removing the monetary policy shocks to consider its role later in this discussion. Hence the policy rule is

$$\hat{i}_t = E_t \hat{\pi}_{t+1} + \theta_d u_t^d.$$

Substituting it into the IS equation, we find

$$\widehat{y}_t^{gap} = \alpha \hat{y}_{t-1}^{gap} + E_t \hat{y}_{t+1}^{gap} + \sigma(1 - \theta_d) u_t^d \ .$$

The output gap admits a solution as AR(1) process:

$$\widehat{y}_t^{gap} = \gamma \hat{y}_{t-1}^{gap} + \frac{\sigma}{1-\gamma}(1-\theta_d)u_t^d = \gamma \hat{y}_{t-1}^{gap} + \tilde{u}_t^d \ ,$$

where $\gamma = 1/2 \pm 1/2\sqrt{1-4\alpha}$, and $\tilde{u}_t^d$ is the rescaled aggregate demand shock.

We can now solve $\hat{\pi}_t$ as a function of $\hat{y}_t$ by taking expectations of its equation for $\hat{\pi}_{t+1}$ and solving it forward in $\hat{y}_t$. We obtain the solution as a system of two equations:

$$\hat{\pi}_t = \frac{\kappa}{1-\beta\gamma}\widehat{y}_t^{gap} + u_t^s, \tag{4}$$

$$\widehat{y}_t^{gap} = \gamma \hat{y}_{t-1}^{gap} + \tilde{u}_t^d. \tag{5}$$

The model features a reduced form Philips curve – i.e. a positive correlation in the data between inflation and slack in the economy – the strength of which depends on $\kappa$, the parameter of the structural Phillips curve. When $\kappa$ is zero the model obtains a flat structural curve and a flat reduced form curve. In such a scenario, the debated flattening of the reduced form Philips curve is due to structural changes in the goods markets or in the firms' pricing mechanisms that have weakened the link between inflation and marginal costs.

It is also interesting to observe that the overall size of the fluctuations in the output gap depends on $\theta_d$, the parameter of the response of the central bank to demand shock. For $\theta_d = 1$ the central bank can completely offset demand shocks and create a flat reduced form relationship between prices and the output gap. The fluctuations in prices would be due to supply shocks and being possibly orthogonal to output – this would correspond to an extreme case of the hypothesis of McLeay and Tenreyro (2020) where the reduced form Philips curve is not present in the post-Volcker data due to the optimal response of the central bank to demand shocks.

## 2.3 Macroeconomic fluctuations and the common trends

How does the description of the business cycle provided above fit into a framework with trends and cycles? Let us first observe that under the parametric restrictions we discussed above the interest rate is a simple function of the output gap in the form

$$i_t = \left(\frac{\gamma\kappa}{1-\beta\gamma} + \frac{1}{\sigma(1-\theta_d)}\right)\widehat{y}_t^{gap} - \frac{\gamma}{\sigma(1-\theta_d)}\widehat{y}_{t-1}^{gap} = \delta_i(L)\widehat{y}_t^{gap}, \tag{6}$$

while inflation expectations are

$$E_t\hat{\pi}_{t+1} = \gamma\widehat{y}_t^{gap}. \tag{7}$$

This formulation of the cyclical components in Equations (4-5), along with Equation (6) can be extended to incorporate unemployment. It fits into an unobserved component model with the state equations for the common cyclical component and the idiosyncratic trends described above, and the observation equation given by

$$\begin{pmatrix} y_t \\ u_t \\ \pi_t \\ E_t\hat{\pi}_{t+1} \\ i_t \end{pmatrix} = \begin{pmatrix} 1 \\ \delta_u \\ \delta_\pi \\ \delta_{E\pi} \\ \delta_i(L) \end{pmatrix} \widehat{y}_t^{gap} + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tau_t^y \\ \tau_t^u \\ \tau_t^\pi \\ \tau_t^i \end{pmatrix}. \tag{8}$$

The empirical specification, that we discuss in the the next section, will follow this representation tightly. However, it expands this stripped down model to incorporate deviations from rational expectations and idiosyncratic 'wedges' that can absorb measurement errors and model misspecification.

## 2.4 The structural limits of the representation

Let us pause now to think about the limits of this representation when interpreted in terms of structural shocks. It is important to stress that the common component that the empirical model will estimate will be able only to capture the bulk of the co-movement among the variables at business cycle frequencies and that its interpretation in terms of fundamental structural shocks may be not always clear-cut. To explain this point, let's consider adding a monetary policy shock to the policy rule:

$$\hat{i}_t = E_t \hat{\pi}_{t+1} + \theta_d u_t^d + u_t^{mp}.$$

Substituting it into the IS equation, we now get

$$\widehat{y}_t^{gap} = \alpha \hat{y}_{t-1}^{gap} + E_t \hat{y}_{t+1}^{gap} + \sigma(1-\theta_d) u_t^d - \sigma u_t^{mp} \ ,$$

and the output gap is

$$\widehat{y}_t^{gap} = \gamma \hat{y}_{t-1}^{gap} + \frac{\sigma}{1-\gamma}(1-\theta_d) u_t^d - \frac{\sigma}{1-\gamma} u_t^{mp} = \alpha \hat{y}_{t-1}^{gap} + \tilde{u}_t^d \, .$$

While this equation still has the same form it had before, the relationship between the policy rate and output gap is not as neat as before since it is not possible to write the demand and the monetary policy shocks as a function of the output gap and its lags (the representation is not invertible). Hence, by estimating a model of the form presented in Equation (8) one would capture the bulk of the common correlation due to common business cycle shocks (and including monetary policy), but may end up dissociating the variable originating the shock, in this case the policy rate, from the estimate business cycle component. We will return to this point later.

# 3 A semi-structural model of trends and cycles

Our empirical framework adopts and generalises the model described in the previous section to capture the joint dynamics of real activity – i.e. output, employment and unemployment rate –, nominal variables – i.e. consumer price inflation and oil prices –, and expectations – i.e. professional forecasts of inflation and output, and consumers' expectations of inflation.

We first introduce our baseline specification that captures the bulk of commonalities at business cycle frequency, in the spirit of Burns and Mitchell (1946), which we estimate on more than one hundred years of US economic history. We then present a specification that includes oil prices in order to understand the residual price dynamics in terms of energy price disturbances. To compare the two specifications, we focus on the post-World War II sample, for which more reliable data are available. For that period, we also perform a stability analysis to assess the potential variation of cycles and trends over time, and its causes.

## 3.1 The baseline model

The primitive measure of the business cycle in the model, and the key measure we focus on is the output gap, $\widehat{y}_t^{gap}$. In the empirical model, it is estimated as an economy-wide stationary stochastic component common to all real variables, labour market variables, inflation, and survey expectations. Its contemporaneous and lagged values are reflected in the price gap via the Phillips curve, and the unemployment gap via Okun's law. These assumptions inform the multivariate restriction that informs the core of the model and allows for the estimation of a common cycle at business cycle frequency.

Following what is standard in the trend-cycle models à la Harvey (1985), we model the output gap as ARMA(2,1) which is the simplest process to display a pseudo-cyclical behaviour.

It can be written in a VAR(1) representation as

$$\widehat{y}_t^{gap} = \rho\cos(\lambda)\widehat{y}_{t-1}^{gap} + \rho\sin(\lambda)\widehat{y}_{t-1}^{gap,*} + v_t \ , \tag{9}$$
$$\widehat{y}_t^{gap,*} = -\rho\sin(\lambda)\widehat{y}_{t-1}^{gap} + \rho\cos(\lambda)\widehat{y}_{t-1}^{gap,*} + v_t^* \ ,$$

where and $v_t$ and $v_t^*$ are uncorrelated white noise disturbances.

In this representation, the cyclical nature of the output gap is in evidence, with the parameters $0 \leq \lambda \leq \pi$ being the frequency, and $0 \leq \rho \leq 1$ the damping factor on the amplitude of the cycle (the process is stationary for $\rho < 1$). $\widehat{y}_t^{gap,*}$ is an auxiliary cycle that allows for the VAR(1) representation.[5] The intuition for the use of the auxiliary cycle is closely related to the standard multivariate VAR(1) representation of univariate AR(p) processes. In fact, the equations can be rewritten as an ARMA(2,1):

$$(1 - 2\rho\cos(\lambda)L + \rho^2 L^2)\widehat{y}_t^{gap} = (1 - \rho\cos(\lambda)L)v_t + (\rho\sin(\lambda)L)v_t^* \ ,$$

where $L$ is the lag operator.

The model estimates the output gap and its reflection on inflation and the labour market, jointly with the long-run trends. Specifically, output is assumed to fluctuate around its potential, which is modelled as a stochastic trend with drift defining the long-run behaviour of GDP:

$$\tau_t^y = \kappa + \tau_{t-1}^y + u_t^{\tau,y} \ . \tag{10}$$

In the spirit of Beveridge and Nelson (1981), it coincides with the long-run forecast of output implied by the model.

Employment and the unemployment rate have their own long-run components defined as a stochastic trend. We denote them as $\tau_t^e$ and $\tau_t^u$, respectively. $\tau_t^u$ is the estimate of the non-accelerating inflation rate of unemployment (NAIRU).

[5] Under the restriction $\sigma_v^2 = 0$, the solution of the model is an AR(2), otherwise an ARMA(2,1).

Table 1: US data and common components

| **Variable name** | **Label** | **Model** | | | **Loads on** | | |
|---|---|---|---|---|---|---|---|
| | | 100y | PW | Oil | *BC* | *EPC* | *Trend* $\pi$ |
| Real GDP | $y_t$ | • | • | • | ✓ | | |
| Unemployment rate | $u_t$ | • | • | • | ✓ | | |
| Employment | $e_t$ | • | • | • | ✓ | | |
| WTI spot oil price | $oil_t$ | · | · | • | ✓ | ✓ | |
| CPI | $\pi_t$ | • | • | • | ✓ | ✓ | ✓ |
| Core CPI | $\pi_t$ | · | • | • | ✓ | ✓ | ✓ |
| SPF: expected inflation | $F_t^{spf}\pi_{t+12}$ | • | • | • | ✓ | ✓ | ✓ |
| UoM : expected inflation | $F_t^{uom}\pi_{t+12}$ | • | • | • | ✓ | ✓ | ✓ |
| Short-term interest rate | $i_t$ | • | • | • | ✓ | ✓ | ✓ |

*Notes:* Data used in the three trend-cycle models discussed in this section: the 120-year sample model (100y), the PostWar (PW) and the model incorporating energy prices (Oil). The columns under 'Model' show, for each model, the variables and the frequencies incorporated in each specification. All data is in levels, except for CPI which is in YoY (%). 'UoM: expected inflation' is the University of Michigan, 12-months ahead expected inflation. 'SPF: expected inflation' is the Survey of Professional Forecasters, 4-quarters ahead expected inflation rate. Data sources and samples are reported in Table 3.

A second structural measure, that is modelled as common across variables is trend inflation, the stochastic trend $\tau_t^{\pi}$. It is estimated as the common trend shared by headline inflation, core inflation, inflation expectations, and the nominal interest rate. By construction, it is the long-run model-based forecast of inflation. The presence of forward expectations, sharing a trend with different measures of inflation, provides multivariate restrictions that inform the estimation. Finally, the nominal rates are also driven by an independent unit root process that can be seen as related to the equilibrium real interest rate.[6]

We also assume that all of the processes have mutually orthogonal stochastic innovations. This is an important assumption for the identification of the unobserved components.

To fit the data, we complete the empirical specification, by introducing several variable-specific components that absorb idiosyncratic shocks, measurement errors, and misspecification which could distort the empirical estimates of the structural relationships. These idiosyncratic

[6]It would be of interest to model the real equilibrium interest rate as connected to output potential growth. However, we leave that as unmodelled in this paper for the sake of simplicity. It is important to stress that the decision on which multivariate relationships to explicitly model, and which others to leave unmodelled, is important and has to be based on the scope of the model as well as on the evaluation of the relative benefits of complexity and parsimony in estimation and forecasting.

components can be seen as 'empirical' wedges capturing the gap between observed data and the assumed structural relationships between variables. In particular, we consider two types of idiosyncratic components: stationary and non-stationary.

Each variable $i$ is modelled as having an idiosyncratic stationary component, $\psi_{i,t}$, which absorbs different sources of idiosyncratic dynamics such as idiosyncratic shocks, non-classic measurement error, differences in definitions, and other sources of noise. These stationary components are modelled as ARMA(2,1) processes, as done for the output gap.

Conversely, non-stationary components are meant to capture persistent time-varying biases in survey data. Agents' expectations can deviate persistently from a rational forecast due to time-varying bias – respectively $\mu_t^{spf,\pi}$ for the professional forecasters' and $\mu_t^{uom,\pi}$ for consumers' expectations. These bias terms are modelled as stochastic random walk components.

Taken together these assumptions imply the following representation of the observed variables that we include in the model (see Table 1 for a summary):

$$\begin{pmatrix} y_t \\ u_t \\ e_t \\ \pi_t \\ \pi_t^c \\ F_t^{spf}\pi_{t+12} \\ F_t^{uom}\pi_{t+12} \\ i_t \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ \sum_{j=0}^{1}\gamma_{2,j}L^j \\ \sum_{j=0}^{1}\gamma_{3,j}L^j \\ \sum_{j=0}^{1}\gamma_{4,j}L^j \\ \sum_{j=0}^{1}\gamma_{5,j}L^j \\ \sum_{j=0}^{1}\gamma_{6,j}L^j \\ \sum_{j=0}^{2}\gamma_{7,j}L^j \\ \sum_{j=0}^{2}\gamma_{8,j}L^j \end{pmatrix} \widehat{y}_t^{gap} + \begin{pmatrix} \psi_{1,t} \\ \psi_{2,t} \\ \psi_{3,t} \\ \psi_{4,t} \\ \psi_{5,t} \\ \psi_{6,t} \\ \psi_{7,t} \\ \psi_{8,t} \end{pmatrix}}_{\text{Common \& Idiosyncratic Cycles}} + \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tau_t^y \\ \tau_t^u \\ \tau_t^e \\ \tau_t^\pi \\ \mu_t^{spf,\pi} \\ \mu_t^{uom,\pi} \\ \tau_t^i \end{pmatrix}}_{\text{Trends \& Biases}}. \tag{11}$$

## 3.2 A model with energy prices

We also consider a second model that adds a common stationary component, which we call the 'energy price cycle' that captures the direct effect of energy shocks on headline inflation. This may be thought of as a way to model empirically the role of energy price disturbances as markup shocks.

The energy price component, $\xi_t^{epc}$, is a stationary stochastic common cyclical component connecting oil prices, inflation, and inflation expectations. It is modelled as an ARMA(2,1) process, as done for the output gap, i.e.

$$\begin{aligned} \xi_t^{epc} &= \rho\cos(\lambda)\xi_{t-1}^{epc} + \rho\sin(\lambda)\xi_{t-1}^{epc,*} + \upsilon_t \ , \\ \xi_t^{epc,*} &= -\rho\sin(\lambda)\xi_{t-1}^{epc} + \rho\cos(\lambda)\xi_{t-1}^{epc,*} + \upsilon_t^* \ , \end{aligned} \tag{12}$$

Therefore, this second model has the following observation equation:

$$\begin{pmatrix} y_t \\ u_t \\ e_t \\ oil_t \\ \pi_t \\ \pi_t^c \\ F_t^{spf}\pi_{t+12} \\ F_t^{uom}\pi_{t+12} \\ i_t \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ \sum_{j=0}^{1}\gamma_{2,j}L^j & 0 \\ \sum_{j=0}^{1}\gamma_{3,j}L^j & 0 \\ \sum_{j=0}^{1}\gamma_{4,j}L^j & 1 \\ \sum_{j=0}^{1}\gamma_{5,j}L^j & \sum_{j=0}^{2}\delta_{5,j}L^j \\ \sum_{j=0}^{1}\gamma_{6,j}L^j & \sum_{j=0}^{2}\delta_{6,j}L^j \\ \sum_{j=0}^{2}\gamma_{7,j}L^j & \sum_{j=0}^{2}\delta_{7,j}L^j \\ \sum_{j=0}^{2}\gamma_{8,j}L^j & \sum_{j=0}^{2}\delta_{8,j}L^j \\ \sum_{j=0}^{2}\gamma_{9,j}L^j & \sum_{j=0}^{2}\delta_{9,j}L^j \end{pmatrix} \begin{pmatrix} \widehat{y}_t^{gap} \\ \xi_t^{epc} \end{pmatrix} + \begin{pmatrix} \psi_{1,t} \\ \psi_{2,t} \\ \psi_{3,t} \\ \psi_{4,t} \\ \psi_{5,t} \\ \psi_{6,t} \\ \psi_{7,t} \\ \psi_{8,t} \\ \psi_{9,t} \end{pmatrix}}_{\text{Common \& Idiosyncratic Cycles}} + \underbrace{\begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&1&1&0&0 \\ 0&0&0&0&1&0&1&0 \\ 0&0&0&0&1&0&0&1 \end{pmatrix} \begin{pmatrix} \tau_t^y \\ \tau_t^u \\ \tau_t^e \\ \tau_t^{oil} \\ \tau_t^{\pi} \\ \mu_t^{spf,\pi} \\ \mu_t^{uom,\pi} \\ \tau_t^i \end{pmatrix}}_{\text{Trends \& Biases}} . \tag{13}$$

We consider two versions of this specification. One in which interest rates systematically respond to the energy disturbances and a second one in which they do not do so (i.e. $\delta_{9,j} = 0$ $\forall j$).

In this specification, oil price enters in levels while inflation is left in rates. Hence disturbances to the energy price cycle affect oil price and the inflation rate. It is worth observing that such a specification in which changes to the level of oil prices directly impact the rate of inflation is compatible with a model in which the demand for energy good is very inelastic (see Gaulier et al., 2023).[7]

It is crucial to note that we do not ascribe a strictly structural interpretation to the energy price component. Instead, it reflects a combination of unidentified structural shocks – oil supply shocks, commodity price shocks, supply chains disruptions, etc – which, as such, does not affect (or only weakly affect) output and the labour market. Hence, by construction, it is orthogonal to the main drivers of the business cycle fluctuations that are captured by the output gap. While orthogonal to the real economy, the energy price component affects oil prices, inflation, and inflation expectations.[8]

### 3.3 Bayesian estimation

The model can be cast in a linear state-space form and estimated with Bayesian techniques, employing an Adaptive Metropolis-Within-Gibbs algorithm (details are provided in Section D of the Online Appendix). We adopt the simulation smoother of Durbin and Koopman (2002) along with the Jarociński (2015)'s modification to condition our estimates of cycles and trends on the full sample.

Data of each variable are normalised by dividing them by the standard deviation of their

[7]To appreciate this point, one can observed that in a standard first order decomposition formula $dlog(CPI) = w_{\text{oil}} \times dlog(P_{\text{oil}}) + [...]$, where $w_{\text{oil}}$ is the oil share. Suppose consumption follows a CES function with elasticity of substitution $\sigma$. Then $CPI^{1-\sigma} = \omega P_{\text{oil}}^{1-\sigma} + (1-\omega)P_{other}^{1-\sigma}$, where $\omega$ is an invariant parameter. The share of oil in total CPI is equal to $\omega(P_{\text{oil}}/CPI)^{1-\sigma}$. With a Cobb-Douglas utility function, $\sigma = 1$, and the share of oil is invariant, and equal to $\omega$. In the case of a Leontiev utility (or close to Leontiev), which is a reasonable assumption for oil, $\sigma$ is close to 0. Hence the oil share varies like $P_{\text{oil}}/CPI$. This means that the relevant elasticity depends on the actual level of the oil prices and is not an invariant parameter any more. By rewriting the elasticity as $dlog(CPI) = \omega d(P_{\text{oil}})/CPI$, the formula gives an invariant semi-elasticity. With a Leontief what is invariant is not the elasticity of CPI with respect to the price of oil, but the semi-elasticity of CPI with the respect to the relative price of oil. This justify a specification in which oil prices enter in level and not in log. An extended discussion on this point is in the Appendix of Gaulier et al. (2023).

[8]The energy price component captures both structural shocks and their transmission through expectations, as for example pointed out by Coibion and Gorodnichenko (2015).

Table 2: Prior distributions

| Name | Support | Density | Parameter 1 | Parameter 2 |
|---|---|---|---|---|
| $\delta$, $\gamma$, $\phi$ and $\tau$ | $\mathbb{R}$ | Normal | 0 | 1000 |
| $\sigma^2$ and $\varsigma^2$ | $(0, \infty)$ | Inverse-Gamma | 3 | 1 |
| $\rho$ | $[0.001, 0.970]$ | Uniform | 0.001 | 0.970 |
| $\lambda$ | $[0.001, \pi]$ | Uniform | 0.001 | $\pi$ |

*Notes:* Prior distribution for the model parameters adopted in estimating the model with US data. All of the priors are uniform over the range of the model parameters compatible with our modelling or weakly informative. Boundaries of the uniform priors ensure that the stochastic cycles are stationary and correctly specified according to the restrictions described in Harvey (1990).

first differences.[9] To deal with missing observations, we employ a Kalman filter approach (see, as a reference, the discussion in Shumway and Stoffer, 1982), and reconstruct the data based on the information available at each point in time. The prior distributions elicited are described in Table 2.

# 4 A century of data

In the empirical analysis we consider the longest spans of quarterly data available for a sample starting in 1901. While for the post-World War II period, most of the series are readily available as produced from statistical offices and the Fed, for the longer sample we need to rely on previous studies that have constructed historical time series, or deal with missing observations when information is not available.

Table 3 describes sources, frequency of available observations, and data treatment. For real GDP, we use the series from Gordon (1986) for the pre-war sample.[10] When data are not available at quarterly frequency, we include them as annual and treat the quarterly

[9]As discussed in Hasenzagl et al. (2022) this normalisation is to set data on a similar scale and provides better mixing in the Metropolis algorithm.

[10]The tables of Gordon (1986)'s 'The American Business Cycle', which have been compiled as an independent project in collaboration with Nathan S. Balk, are available on the website of the NBER. This data set is the only existing source for the pre-1947 quarterly data, as NIPA quarterly data series do not exist before 1947. The dataset includes the components of GDP back from 1941 to 1919 and the quarterly real GDP back to 1875.

Table 3: Data and Transformation

| Variable | Transf. | Frequency | Period | Source |
|---|---|---|---|---|
| Real GDP | Levels | Q | 1901-1946 | Gordon (1986)'s NBER Tables |
| | | | 1947-2023 | FRED |
| Employment | Levels | A1901, Q1948 | 1901-2023 | Haver Analytics |
| Unemployment Rate | Levels | A1901, Q1929 | 1901-2023 | Haver Analytics |
| Oil Price | Levels | Q1946 | 1946-2023 | Haver Analytics |
| Inflation | YoY | A1914, Q1921 | 1914-2023 | Haver Analytics |
| Core Inflation | YoY | Q1957 | 1957-2023 | FRED |
| Consumers Exp. Inflation | Levels | Q1978 | 1978-2023 | University of Michigan |
| SPF Exp. Inflation | Levels | S1946, Q1983 | 1946-1983 | Livingston Survey |
| | | | 1984-2023 | SPF Philadelphia Fed |
| Nominal short term rate | Levels | A1901, Q1954 | 1901-1954 | Officer (2024)'s Measuring Worth |
| | | | 1954-2023 | FRED |

*Notes:* The table lists the macroeconomic variables used in the empirical model. 'Consumers Exp. inflation' is the University of Michigan, 12-months ahead expected inflation rate. 'SPF Exp. Inflation' is the Survey of Professional Forecasters, 4-quarters ahead expected CPI inflation rate. The oil price is the West Texas Intermediate Spot oil price ($ per barrel).

observations as missing data. This is the case for employment, unemployment and inflation. For the Survey of Professional Forecasters 1-year ahead expected inflation, we concatenate the semi-annual Livingston survey starting in June 1946 with the quarterly Philadelphia FED SPF series published from Q1-1984.

Lastly, for the nominal interest rate, we employ the quarterly federal funds rate from 1954. For the earlier period we use the annual short-term rate of Officer (2024), which is also adopted by the Macrohistory Database of Jordà et al. (2019).[11] It is constructed from the short-term lending or borrowing rates of surplus funds – i.e. call loan –, that is, funds that are considered in excess by the lending institution and are required for immediate temporary use by brokers.[12]

[11]The database is available on the website of the Macrohistory Project.

[12]Specifically, as reported by Officer (2024): "Surplus Funds are available from 1857-present and this information is obtained from the Federal Reserve. From 1857-1954, it was in the form of a call loan. From 1955-present, it is in the form of federal funds. [...] For a consistent series, the change in concept (call loan to federal funds), as well as changes in measure within a concept, are smoothed via linking. Thus the contemporary and consistent series are identical from 1955 onward but not earlier." Data on the Annual average of Federal Funds (FF) is available on the Fed Board website.

# 5 One hundred and twenty years of business cycles

How well can a stylised linear model with fixed parameters fit the U.S. business cycles since the beginning of last century and over a span of time that includes multiple recessions, two world wars, the Great Depression, and the Great Recession? Perhaps surprisingly, quite well.

In this section, we discuss the empirical results for the longest sample, spanning the period 1901-2019. For this sample, given the non-availability of some variables, we only study a baseline specification and discuss a few key results, which we will then explore further for the post-World War II period.[13]

Let us summarise results upfront:

1. Our linear model captures well the business cycle regularities of the U.S. data by providing a measure of the output gap which is coherent with the NBER recession dates, and mostly in line with the official estimates of slack in the U.S. economy by the Bureau of Economic Analysis.

2. The output gap is reflected into prices, with recessions exercising downward pressure on prices and expansion pushing up inflation. Also, the labour market cyclical components comove with the output gap, albeit featuring some instability possibly due to structural changes over the decades.

3. The cyclical component of the short-term interest rate is almost entirely driven by the common component of the business cycle. This points to interest rate fluctuations being largely driven by common shocks which we interpret as being the result of the systematic component of monetary policy. However, there are periods of idiosyncratic fluctuations, reflecting policy that deviates from the historical norm.

We return to a more detailed analysis of the post-1960 period in the next section.

[13]The full set of results is reported in Section A.1 of the Online Appendix.

## 5.1 The business cycle

Let us focus on the estimate of the business cycles by analysing the output gap, the cyclical component of unemployment, inflation and interest rate cycles as well as the inflation trend.

Our estimate of the output gap is indicated in blue in the upper panel of Figure 2. As illustrated in the model, this is the common cyclical component of output, unemployment and employment. The latter acts as a 'primitive' measure of business cycles, to which all the other variables respond, possibly with lags. The yellow is the part of the cycle which is output specific, capturing disturbances affecting only output, measurement errors or absorbing different forms of model misspecification. The bottom panel reports the same decomposition for unemployment.

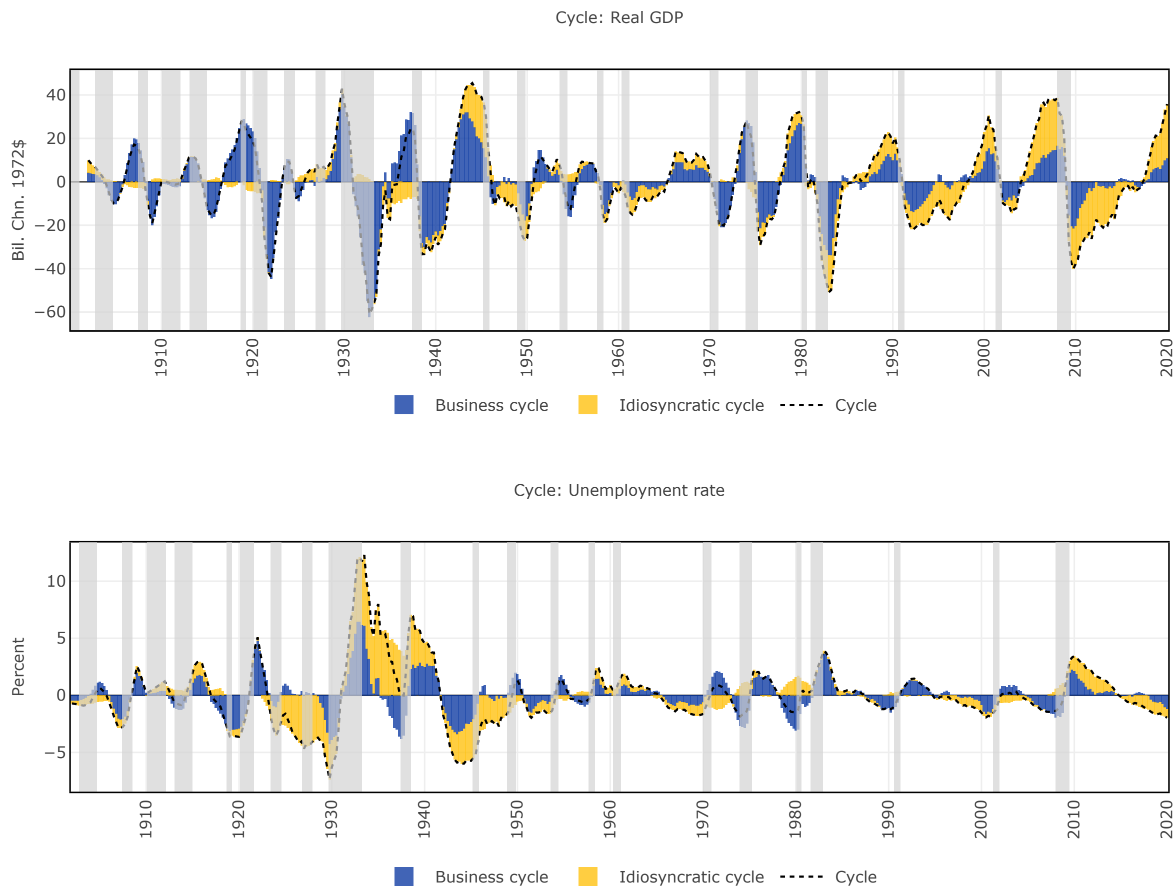
The first observation is that the post-World War II business cycle is less volatile than in the pre-war period. The volatility of the output gap is reflected in the cyclical component of unemployment which is mostly explained by our definition of the Okun's law (blue component). Notice that the unemployment cycle lags the output gap, due to the econometric specification that includes both current and past realisations of the output gap.

The period until the end of Second World War features a highly volatile cycle: first driven by the 1920s expansion, followed by the Great Depression, and finally by the war. Unemployment and output have a large common cycle but also periods of idiosyncratic dynamics until the end of the forties, likely to be the result of structural changes in the labor market due to the exceptional circumstances of the Great Depression, and then the war effort. A tighter relation between the output gap and unemployment emerges since the fifties from when we also detect a decline in cyclical volatility.

## 5.2 The Phillips curve

Let us now comment on the inflation results. Figure 3 shows the cycle (upper panel) and the trend (lower panel). The blue component of the cycle is what we interpret as the Phillips

Figure 2: Historical decomposition of real GDP and unemployment

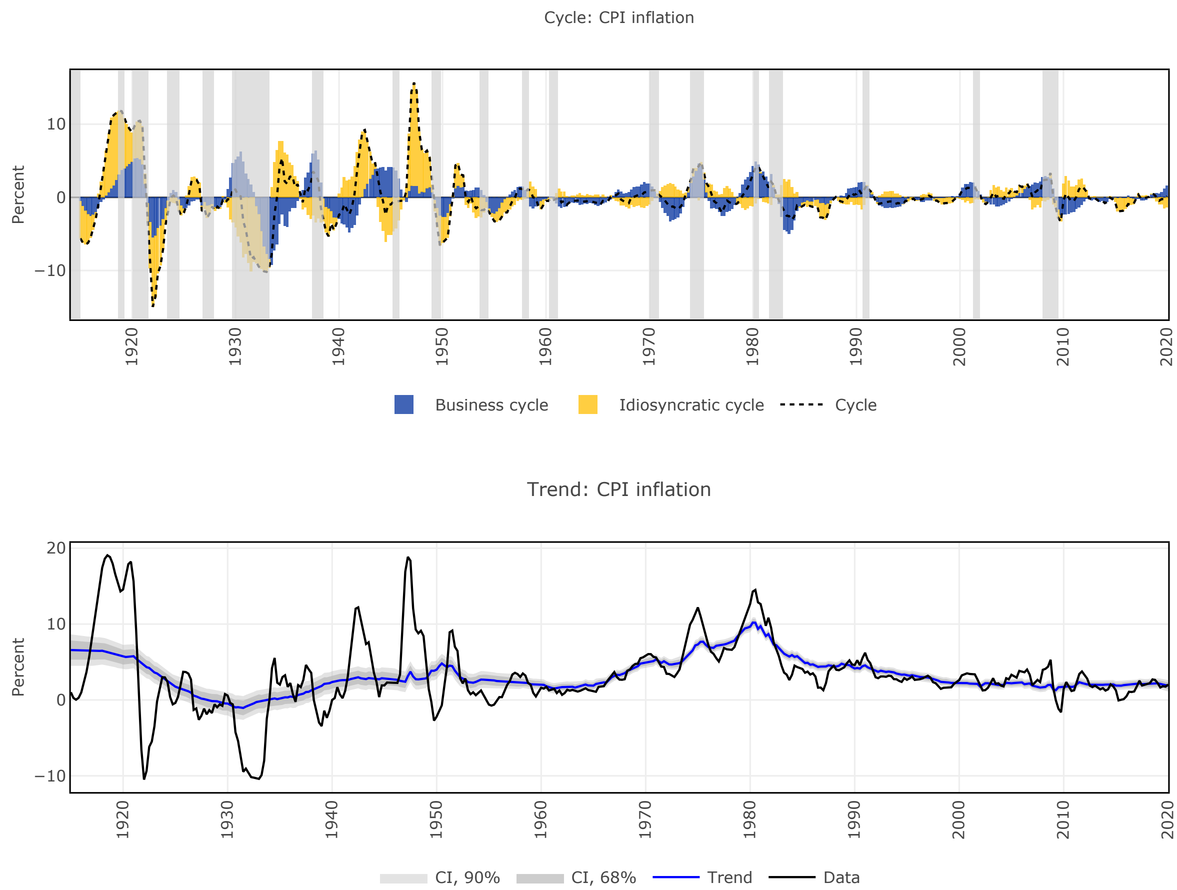



*Notes:* The chart shows the historical decomposition of the cycles of output and unemployment. The chart also reports the business cycle (in blue), and idiosyncratic cycle (in yellow). The model is estimated over the sample 1900-2019.

curve, i.e. the positive relationship between cyclical output and inflation. Again, cyclical volatility is higher pre-1950 and more idiosyncratic.

Until 1921, the US experienced high inflation, driven by the war economy and its aftermath. The post-World War I expansion of the economy lasted until 1920 and pushed inflation up. The model attributes part of the increase in cyclical inflation to a sizeable idiosyncratic component as it does for unemployment. Given the persistence of inflation during those years (from December 1916 to June 1920 annualised inflation increased 18.5% with a cumulative increase of 80% according to the US Bureau of Labor Statistics, 2014), the model attributes part of the surge of inflation to the the trend (lower panel). In June 1920, inflation started

Figure 3: CPI inflation trend and cyclical component




*Notes:* The chart shows the cycle decomposition (top) and common trend (bottom) of CPI Inflation, with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model. The model is estimated over the sample 1900-2019.

falling in association with the recession of the early twenties. The recession featured a significant drop in output and rise in unemployment, but the deflation associated to it was exceptional and cannot entirely be explained by the drop in activity. According to the U.S. Bureau of Labor Statistics, the CPI dropped by more than 20% from June 1920 to September 1922, a volatility that is unique in the sample considered and that the model again attributes to an idiosyncratic factor.[14]

The following years, until the great depression, were years of tight monetary policy which combined the obligations under the gold standard and the 'real bill' principle followed by

[14]See the information provided by the U.S. Bureau of Labor Statistics.

the Federal Reserve. Inflation remained volatile but around a lower average as it is showed by the estimated trend. Higher volatility characterised the period starting with the Great Depression, and until the 1951 when the Treasury-Fed Accord established the end of the Fed's peg to the short-term Treasury bill and the separation between monetary policy and debt management. Overall, higher inflation volatility in the first half of the 20th century is read by the model as being the result of the business cycle (Phillips curve), idiosyncratic factors and changes in trends reflecting different regimes: peace, war and monetary policy.

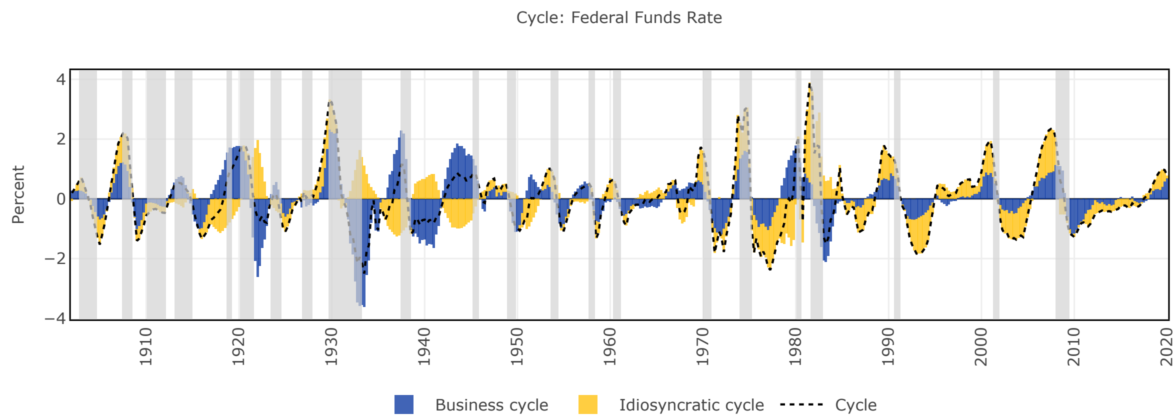
Since the Treasury-Fed Accord, the inflation cycle becomes less volatile and closely matches the Phillips curve component, possibly due to improved monetary policy. The 1950s are a period of stability. Historical evidence attributes this to a systematic response of monetary policy to demand driven inflation (see Romer and Romer, 2002, on this point) which our model captures in the output gap component of inflation (blue area). This is also reflected in the stability of the trend component until the mid-sixties. Since then and until the Volcker's disinflation, trend inflation drifts upward while cyclical inflation tracks the output gap. Stability in trend and cyclical inflation returns in the nineties but is again challenged after the financial crisis. We will comment in more details the decade of inflation stabilisation and the post financial crisis period.

At this stage, let us stress that the model points to a sizeable cyclical component of inflation, reflecting negative correlation with our measure of the slack in the real economy, a feature which would have been obscured if we had not included an idiosyncratic wedge in the model of the cyclical component and subtracted a time-varying trend.

### 5.3 The short-term interest rate

Additional insights can be gained by the analysis of the cyclical component of the short-term interest rate (Figure 4). The model explains a large part of the interest rate by the output gap (blue area). This suggests that monetary policy has been responding systematically to demand driven inflation for the whole sample, pointing to a continuity in Federal Reserve's

Figure 4: Short-term interest rate cyclical component



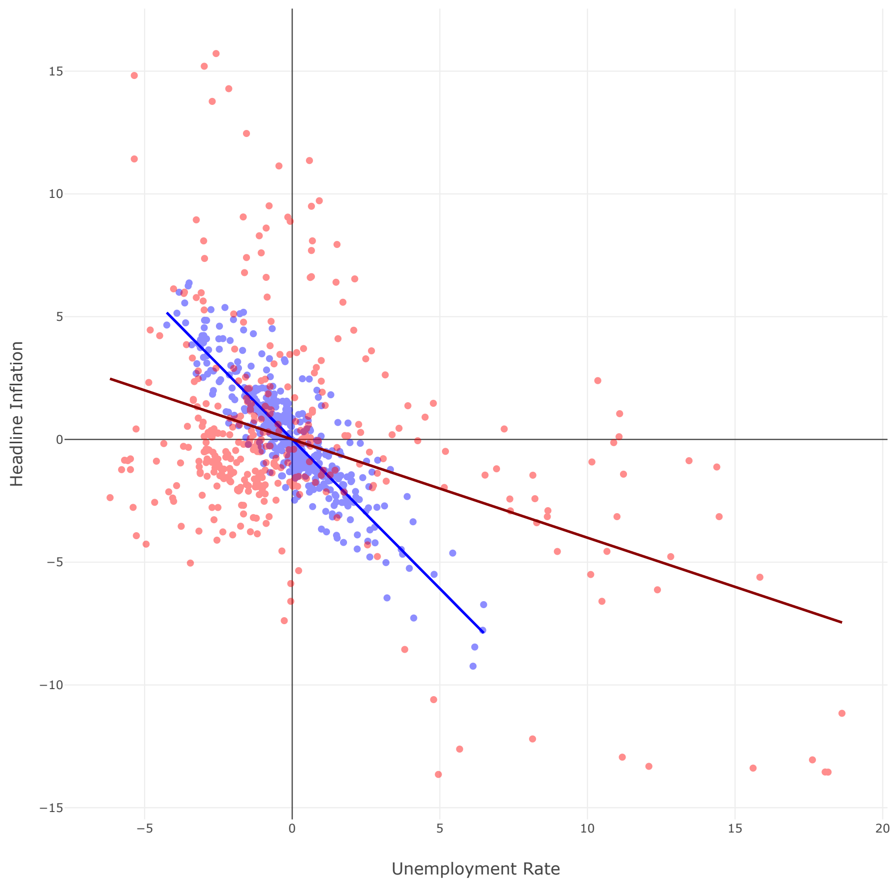

*Notes:* The cyclical component of the short-term interest rate. The chart reports the business cycle (in blue), and idiosyncratic cycle (in yellow). The model is estimated over the sample 1900-2019.

monetary policy (see Bernanke, 2023, for an historical reconstruction of the events) although some idiosyncratic dynamics appear to be relevant occasionally. This reflects deviation of policy from historical norm and becomes sizeable, in particular, between the mid-seventies and the mid-eighties, reflecting out-of-norm loose and then tight monetary policy. In the last part of the sample, out-of-norm tight interest rate policy is due to the zero lower bound constraint.

These results can be interpreted through the lenses of the toy model we presented in Section 2. A large positive correlation between cyclical variation in nominal and real variables reflects the systematic response of interest rate to demand driven cyclical inflation, which itself produces large co-movements between the interest rate cycle and the output gap. The model, not surprisingly, leaves some dynamics unexplained given the presence of multiple shocks and possible non-linearities at the zero lower bound.

The importance of the Phillips curve for inflation is usually expressed in terms of its 'steepness', which is the fitted slope of the empirical negative relationship between the contemporaneous level of inflation and unemployment. In contrast, our model estimates a dynamic Phillips curve and Okun's law that capture, respectively, the components of inflation

Figure 5: Reduced form Phillips curve




*Notes:* This chart plots the business cycle component of CPI inflation against the business cycle component of the unemployment rate (blue dots) and the corresponding bivariate linear regression line (blue line). The chart also plots demeaned CPI inflation against the demeaned unemployment rate (red dots) and the corresponding bivariate linear regression line (red line). The model is estimated over the sample 1900-2019.

and unemployment which are explained by the output gap and its lags. To provide intuition, in Figure 5, we compare the values of demeaned unemployment and inflation (red dots) along with their fitted OLS lines, with the model-based estimates of the values of inflation and unemployment that reflect the common business cycle variation (blue dots) and their fitted slope. With a slight abuse of language we can call the line through the red clouds of points the reduced form Phillips curve and that through the blue cloud the model based Phillips curve.

Two remarks are in order. First, the blue cloud of actual data is more dispersed than the red cloud of values that the model attributes to the common business cycle component. This illustrates the ability of the model to isolate the correlation between unemployment and inflation by removing the variation in the data explained by energy price disturbances and idiosyncratic components. Second, both lines are negatively sloped but the blue line is steeper. The steepness of the reduced form Phillips curve using the model-fitted components is $-1.22$. That compares to the value estimated from the actual data which is $-0.40$ (solid red line). This highlights how other dynamic components that affect prices but not unemployment can both weaken and distort the estimates of the Phillips curve.

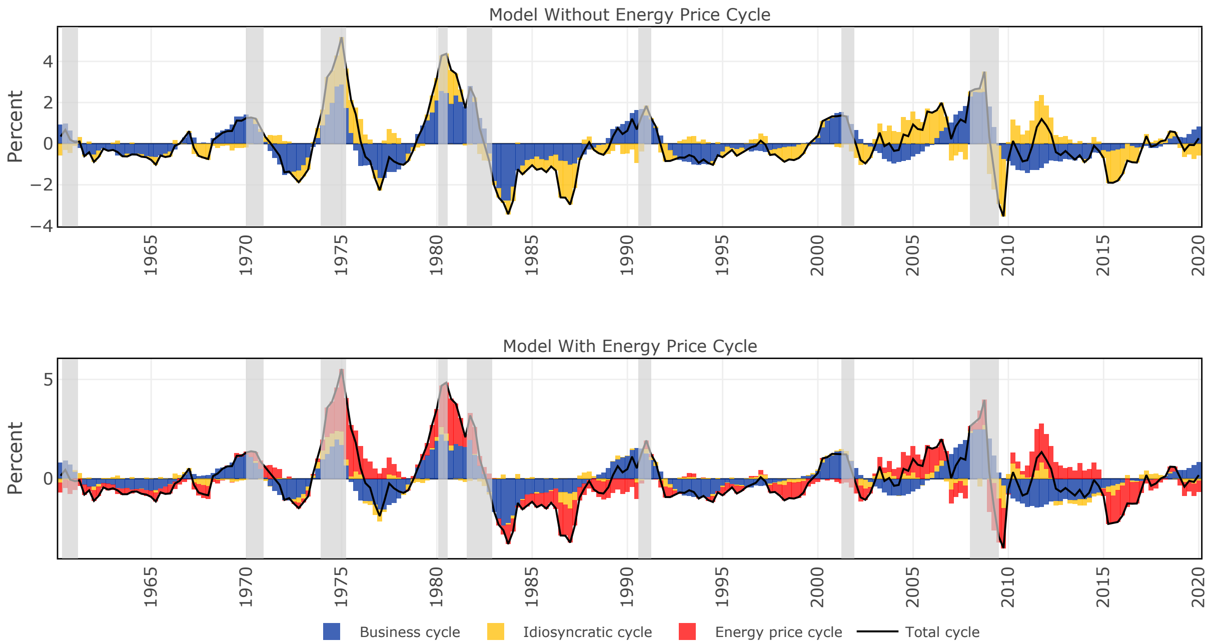
# 6 Postwar Phillips curve

How does the model stand up to a more complete set of data coming from a more stable period of the economic history of the US? We now zoom in the sample starting in 1960 to provide a more detailed analysis. We do this by estimating two models, one which is identical to that of the previous section and one in which we introduce a separate oil cycle as a component of inflation. The purpose of including the oil cycle is to investigate whether part of the idiosyncratic cycles which we described in the last section can be attributed to commodity price variation or factors correlated with commodity prices.[15]

As described earlier in Section 3, the expanded model includes an energy price cycle which is identified as being correlated with all nominal variables in the system but orthogonal to the real variables. While we cannot have a structural interpretation of this component (see discussion in Section 2.4), the assumption of orthogonality is empirically a convenient assumption that can help capturing the role of a combination of economic disturbances that correlated with oil prices and to which monetary policy does not respond to. In the model presented in Section 2, when monetary policy does not respond to supply shocks, the latter become orthogonal to output. In line with the model, we interpret the energy price cycle

[15]While in this section we only report some key results, Sections A.2, A.3, and A.4 in the Online Appendix report the decompositions for all the variables and the models discussed in this sections.

Figure 6: CPI inflation cyclical components




*Notes:* This chart shows the historical decomposition of CPI inflation cycles, comparing the model without (top) and with (bottom) energy price cycle. The chart reports the business cycle (in blue), the energy price cycle (in red), and idiosyncratic cycle (in yellow). The models are estimated over the sample 1960-2019.

estimated by the model as a stationary component of oil prices generated by a convolution of shocks to which monetary policy may not respond to. This interpretation is confirmed by the empirical results.

More broadly, given our identifying restrictions, the oil cycle reflects fluctuations which are determined in the world market, expectation driven fluctuations (see Coibion and Gorodnichenko, 2015), or any form of model misspecification which is correlated with the cyclical dynamic of oil and only weakly impact real variables. The idiosyncratic component captures remaining unexplained features of our trends-cycles decomposition.

## 6.1 The role of energy price fluctuations in inflation

Figure 6 plots cyclical inflation for the two versions of the model: without the oil cycle (upper panel) and with oil (lower panel). Quite starkly, the idiosyncratic spikes of inflation of the mid-seventies and early eighties are now captured almost entirely by the oil cycle. In those years, inflation increased more than what can be explained by the positive output gap and the model attributes this to the energy cycle. At the same level of slack, there is a higher value of cyclical inflation but this cannot be attributed to a shift in demand. Loosely speaking this translates occasionally into a vertical, reduced-form Phillips curve as in some of the high inflation episodes discussed in the Introduction.

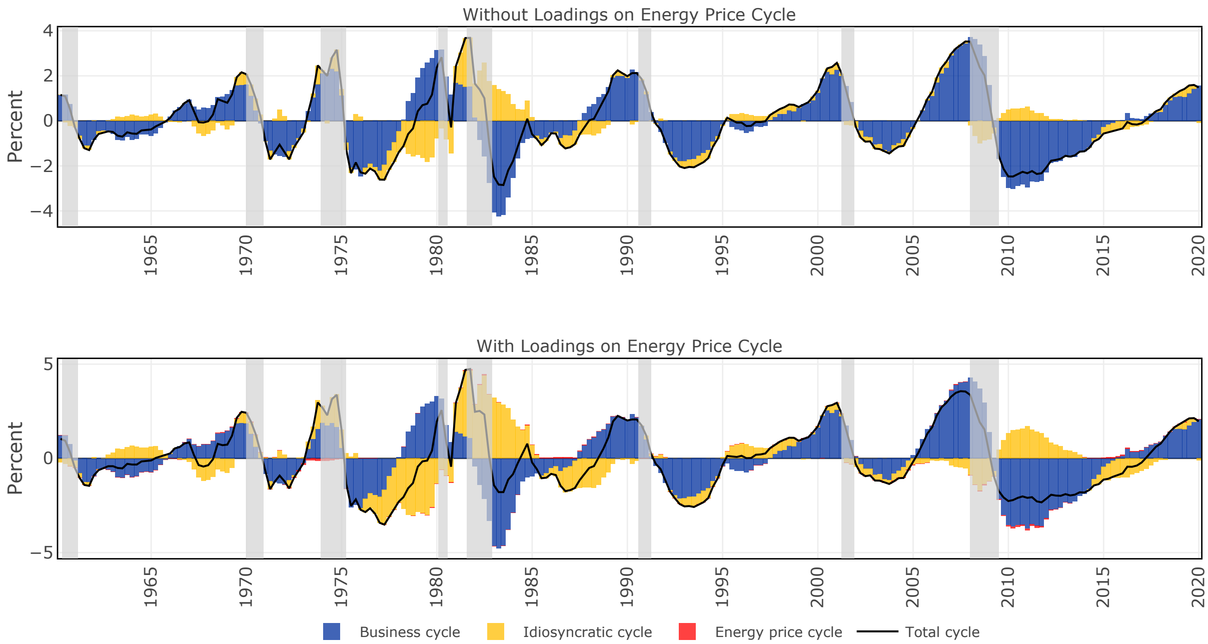
In the following years, the effect of the oil cycle occasionally moves in the opposite direction of the output gap. Interestingly, this explains why, post-financial crisis, inflation did not decline in line with the weak cyclical performance and why, as the economy recovered, it did not bounce back: the disinflation and inflation puzzles (see Hasenzagl et al., 2022, for a discussion).

## 6.2 The policy rule and energy fluctuations in prices

Let us now examine how energy prices impact the federal fund rate (Figure 7) and hence the policy rule. The results with and without oil are almost identical, indicating that the oil cycle is not associated with the federal funds rate cycle. In both cases, in line with what we have seen with the long sample, a large part of the cyclical variation of the federal funds rate is associated with the output gap while there is a large unexplained residual in two periods: 1975-1985 and post-Global Financial Crisis. In the first period, we have the exceptionally low interest rate under Arthur Burns and the exceptional tightness under Paul Volcker while after the Global Financial Crisis, the Zero Lower Bound (ZLB) constrained interest rate to be exceptionally high, given the level of inflation and the output gap. We interpret these large wedges as the effect of policy shocks and deviations from linearity.

Again our model uncovers a large systematic component in cyclical interest rate but also points to periods in which monetary policy deviates from the norm. In the next section we unpack further the components of the model to obtain a better understanding of the stability of the model.

Figure 7: Federal funds rate cyclical component




*Notes:* The chart shows the historical decomposition of the cyclical component of the federal funds rate, as computed by the model which includes energy price cycle. In the top panel, the short-term rate does not load on the energy price cycle, while it does in the bottom panel. The chart also reports the business cycle (in blue), the energy price cycle (in red), and idiosyncratic cycle (in yellow). The models are estimated over the sample 1960-2019.

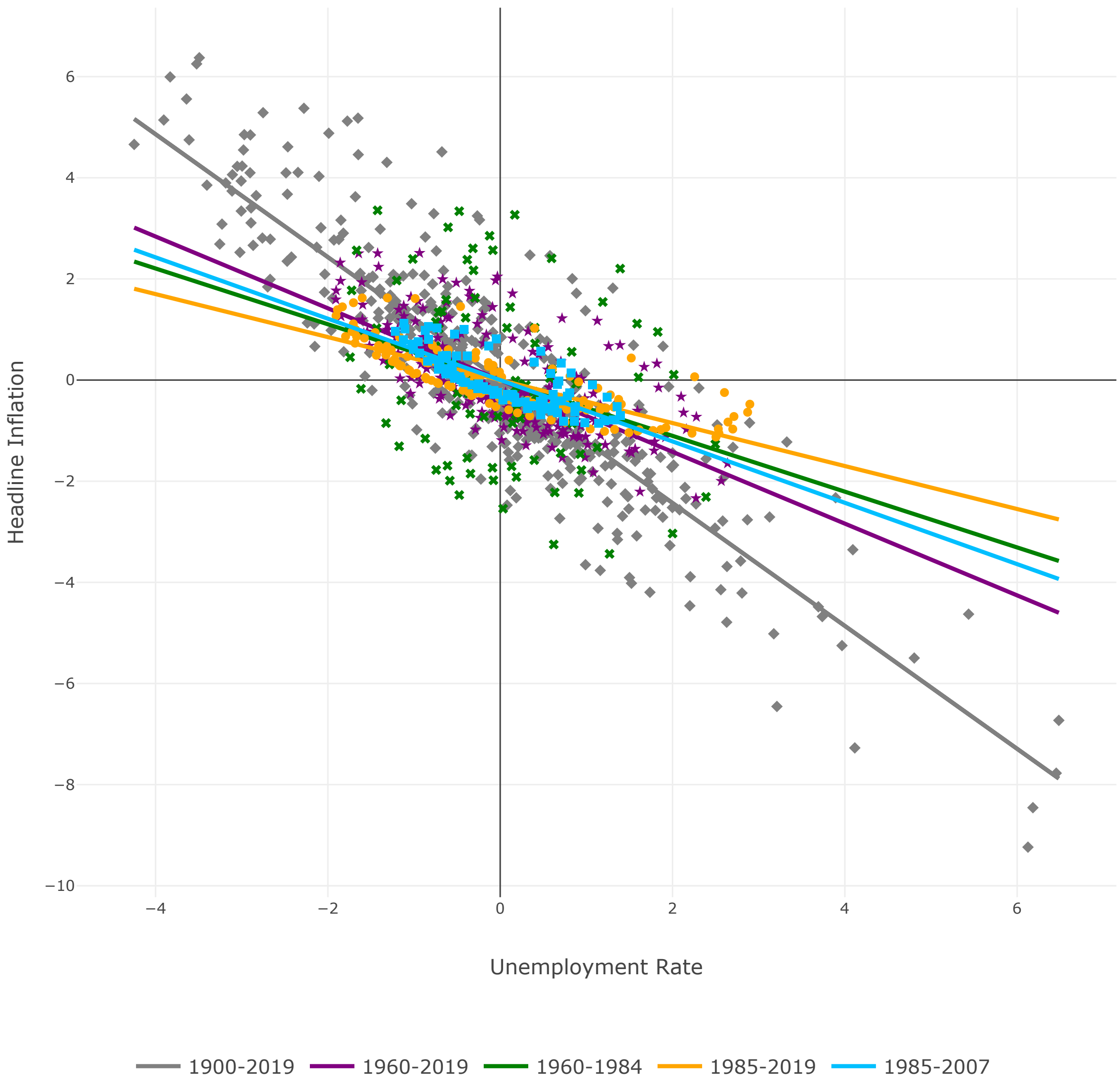
# 7 Stability analysis

We now move to assess the stability of the model estimates across sub-periods. In particular, we consider the periods 1960-1984, 1985-2015 and 1985-2007, as well as the estimates from the full sample and the sample post-1960. We do that by comparing estimates from the model that incorporates oil prices and with the FFR not responding to the energy price cycle for all the post-1960 sample, and the baseline model for the sample starting in 1901.

## 7.1 The reduced form Phillips curve

Let us start by plotting the least squared lines fitting the estimated gap-driven cyclical inflation and cyclical unemployment for the full sample and the three sub-samples (Figure 8).

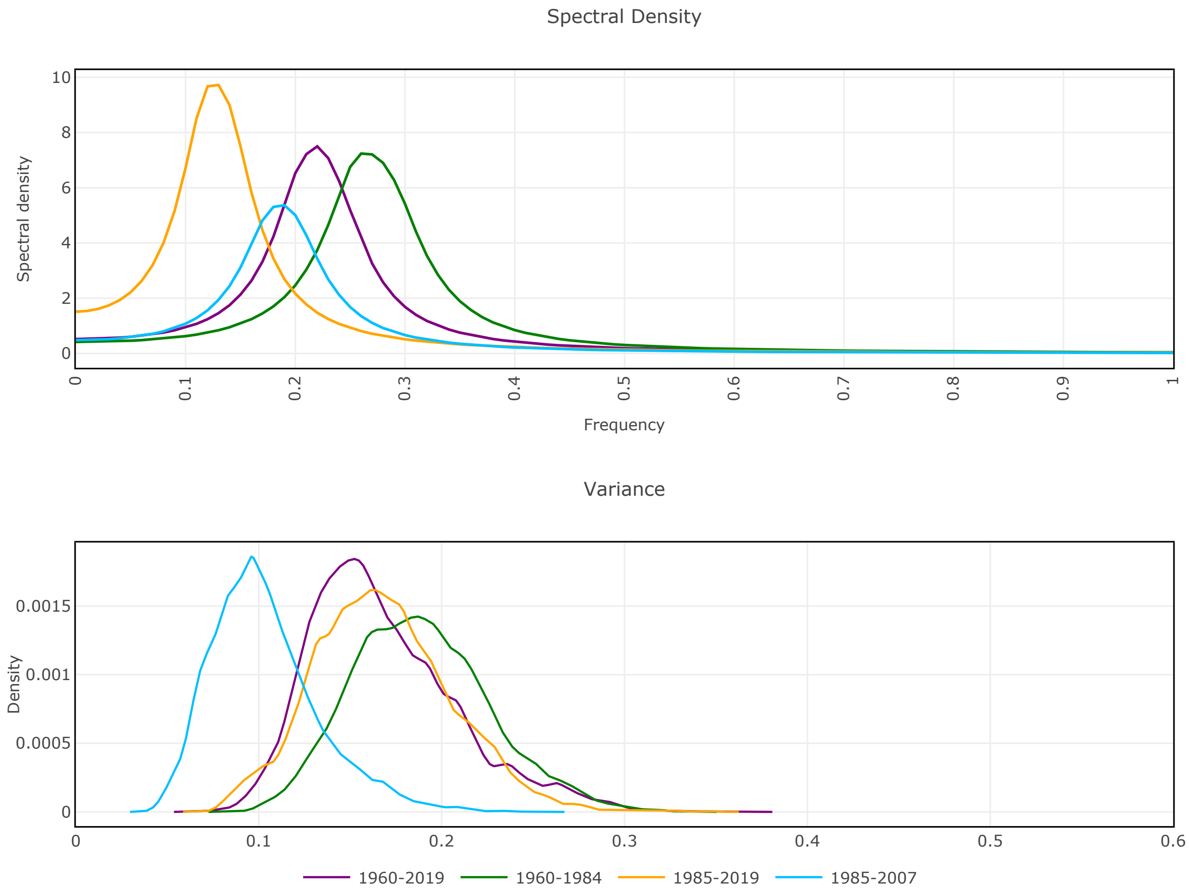
Figure 8: Subsample analysis of the reduced form Phillips curve




*Notes:* This chart plots the business cycle component of CPI inflation against the business cycle component of the unemployment rate and the corresponding bivariate linear regression line for the samples 1900-2019, 1950-2019, 1960-1984, 1985-2015 and 1985-2007.

We also include the long sample 1900-2019 as comparison. In all cases, the Phillips curve is negatively sloped. The largest slope, −1.22, is for the period 1900-2019 due to the extreme shocks of the wars and the great depression. Post-1960, estimates are quite close to one another pointing to a relative stability of the inflation-unemployment relationship once the

Figure 9: Spectral density and variance of the business cycle component




*Notes:* The chart shows the spectral density (top) and the variance (bottom) of the business cycle component for the samples 1960-2019, 1960-1984, 1985-2019, and 1985-2007.

cycle is cleaned by the oil component. For the full sample the slope of the Phillips curve is -0.71.

## 7.2 The model parameters: policy or luck?

How stable have the business cycle regularities been? To answer this question, we look at three statistics: (i) the spectral densities of the output gap; (ii) the posterior densities of the estimates of the variance of the cyclical shock (the reduced form shock driving the output gap), and (iii) the posterior densities of the estimates of the loadings for all the variables included in the model.

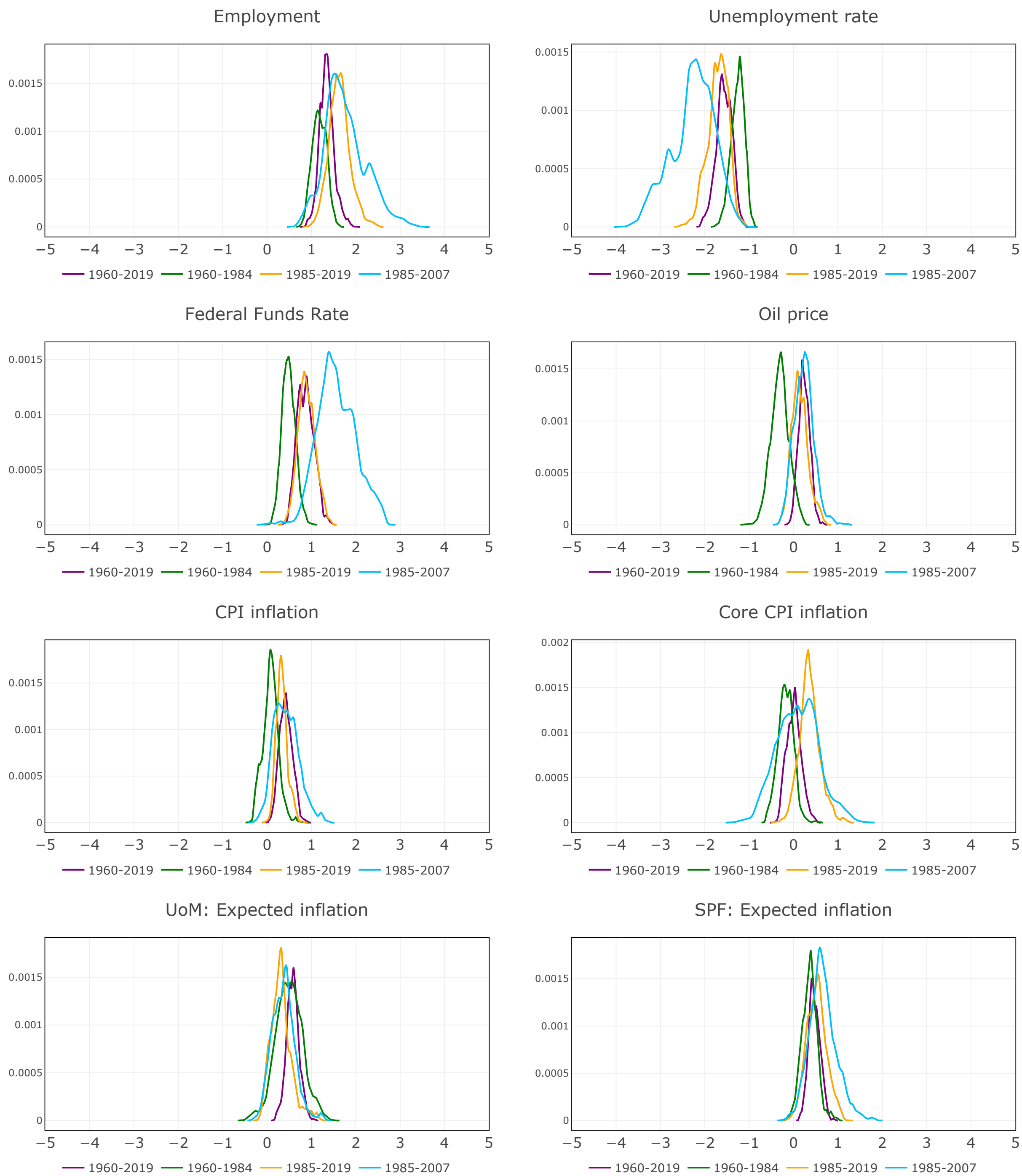
Figure 9 shows the spectral densities of the cycle (upper panel) and the posterior density of the variance of the disturbance of the output gap cycle, (bottom panel) for the sub-samples. The charts indicate that the Great Moderation sample is characterised by the lowest volatility, as shown by both the area under the spectrum (which is equal to the variance of the output gap in a given period) and the mean of the distribution of the variance of the cyclical stochastic disturbances.

The peak of the spectrum is at frequency 0.18, corresponding to a periodicity of just above eight years and in line with the commonly accepted definition of business cycle. The 1960-84 cycle, by contrast, has the shortest periodicity of just below six years. Interestingly, the longer periodicity – of over 10 years – is that of the 1985-2019 sample, reflecting not only the occurrence of few recessions during the Great Moderation, but also the long expansion of the post global financial crisis.

Examining the distribution of the contemporaneous coefficients of the output gap on all the variables included in the model helps deepening our understanding. Their posterior densities are reported in Figure 10. The key result is that the coefficients of the business cycle on CPI inflation and inflation expectations are very stable over time confirming again the main finding of this paper, i.e. the stability of the reduced form Phillips curve relation. Interestingly, this is not the case for the coefficient on interest rate and the unemployment rate.

The coefficient of the federal funds rate is the largest in the Great Moderation and the lowest in the period 1960-84 which suggests that monetary policy was more aggressively responding to the cyclical component of inflation (the component driven by the output gap) in 1985-2007 than in other periods. Similarly, cyclical unemployment was more responsive to the output gap during the Great Moderation than in other periods, most likely as the result of a more flexible labour market. To interpret this finding, we must consider that the model estimates a declining unemployment trend during this period (see Section B in the Online Appendix). Combining this, with the finding by the literature that labor hoarding by firms

Figure 10: Posterior distributions of the coefficients for output gap




*Notes:* The chart shows the posterior distributions for the coefficients of the contemporaneous response of individual variables to output gap for the samples 1960-2019, 1960-1984, 1985-2019, and 1985-2007.

became less important, it is not surprising that the response of cyclical unemployment to cyclical shocks became larger (see Galí and Gambetti, 2019 for a discussion on this point).

In sum, our analysis uncovers three facts about the Great Moderation. First, once the trends are accounted for, the labour market responded more strongly to business cycle fluctuations. Second, the policy rate was more responsive to the slack of the economy and hence to cyclical inflation. Third, the overall volatility of the cycle was subdued.

Economists have been divided between those explaining the Great Moderation as the consequence of 'good luck', i.e. a decline of the volatility of the shocks hitting the U.S. economy, and those attributing it to 'good policy', i.e. an improved macroeconomic framework including inflation targeting and in general a focus on price stability (see Stock and Watson, 2002). While our analysis cannot lead us to any definite conclusion towards either the 'policy' or 'luck' hypothesis since we don't identify the shocks structurally, the facts that we have uncovered suggest that the lower volatility of the Great Moderation is likely to be explained by structural changes in the labour market and by the policy response to those changes.

It is important to stress that the differences in cyclical volatility over sub-samples does not contradict the key result of our analysis which point to a stable relationship between the slack in the economy and inflation as can be seen by the stability of the inflation coefficients to output gap. Our results are in line with the prediction of the stylised model in Section 2 according to which policy acts as a stabiliser of the total amount of volatility in the economy but the correlation between the output gap and inflation remains stable.[16]

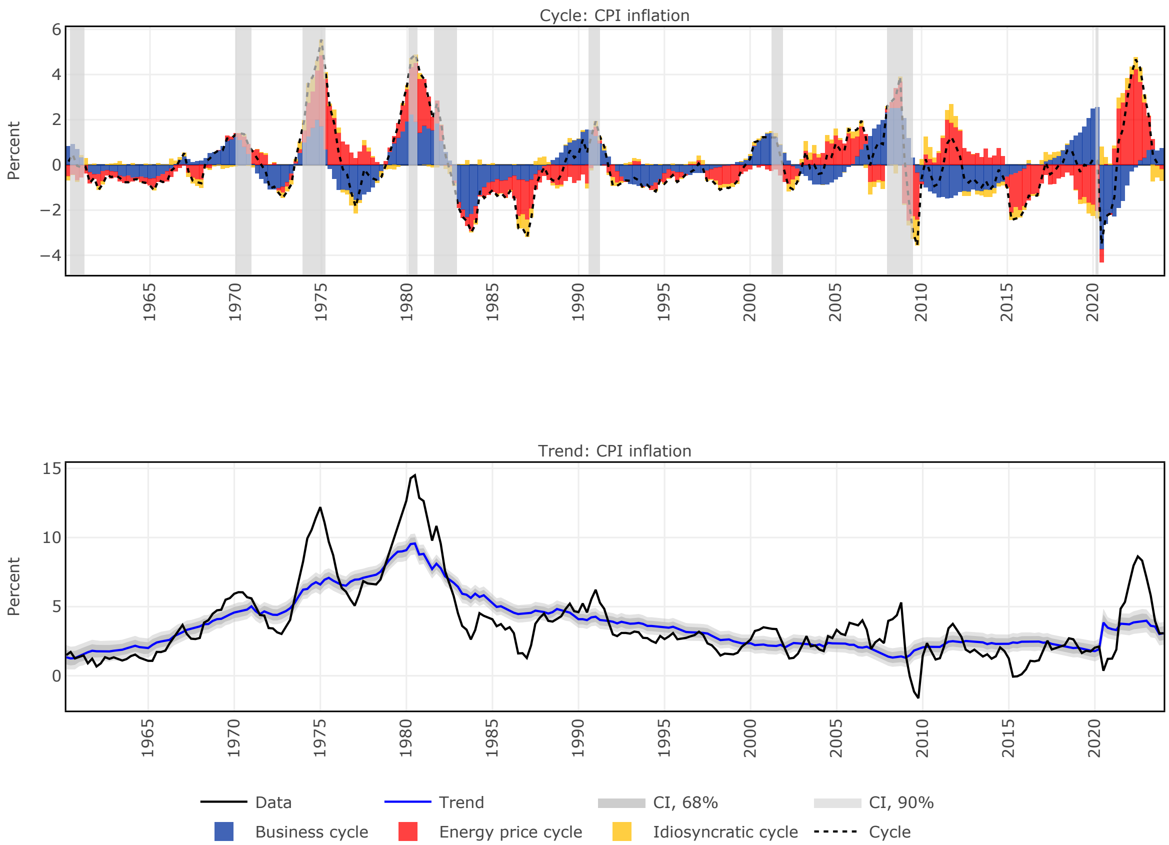
# 8 Inflation and interest rate during COVID

We conclude our discussion by analysing the pandemic and post-pandemic period through the lens of the model that includes oil prices, and with the parameters estimated on the pre-COVID sample, 1960-2019.[17] This provides a reading of how, the model informed by the pre-pandemic regularities can understand the inflation, interest rates, and output dynamics

[16]To appreciate this finding, we must consider that, in contrast with the stability of the coefficient linking the output gap to inflation, the coefficient linking the oil cycle to inflation is unstable over sub-samples (see Section B in the Online Appendix).

[17]This choice is motivated by the high volatility of the last part of the sample, which is likely to be an outlier (see Lenza and Primiceri, 2022, for a discussion).

Figure 11: CPI inflation trend and cyclical component




*Notes:* The chart shows the cycle decomposition (top) and common trend (bottom) of CPI inflation, with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model. The model is estimated over the sample 1960-2019, while the decomposition is performed over the extended sample 1960-2023.

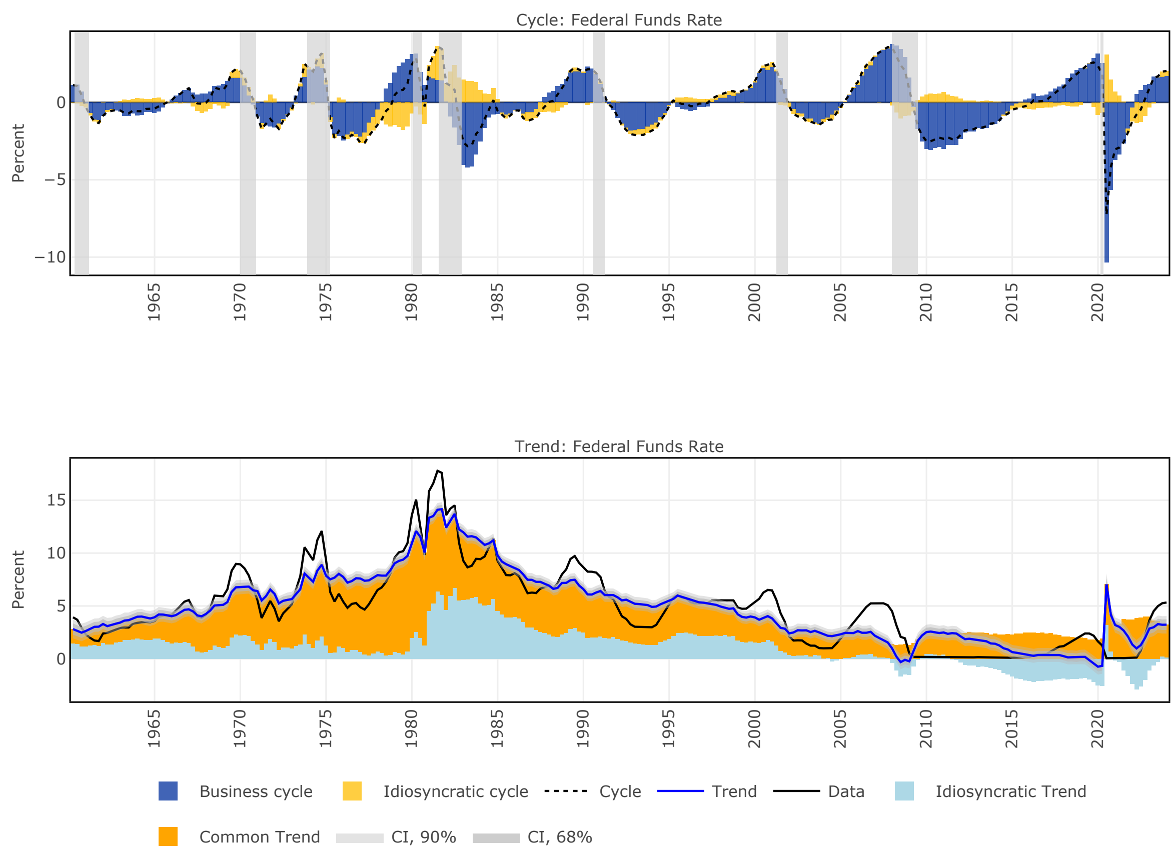
of the last few years.[18]

First, let's focus on inflation (Figure 11). The model attributes the sharp decline in inflation during the pandemic period to the extraordinary negative readings of the output gap, which becomes mildly expansionary only in late 2021. However, the model explains most of the surge in inflation as due to the energy cycle and an increase in trend inflation, reflecting a rise in inflation expectations. Overall, the trend and energy components are the main contributors to the initial spike in inflation and its subsequent decline, while the cyclical

[18] A full set of results is provided in Section A.5 of the Online Appendix.

component plays a very minor role.

Figure 12: FFR trend and cyclical component




*Notes:* The chart shows the cycle decomposition (top) and the trend (bottom) of the federal funds rate, with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model. The trend is decomposed as common trend between nominal variables and idiosyncratic trend. The model is estimated over the sample 1960-2019, while the decomposition is performed over the extended sample 1960-2023.

A decomposition of the federal funds rate leads to a number of interesting observations (Figure 12). Let us begin with the chart below, which shows a breakdown of trend inflation into the trend inflation (orange) and the trend specific to the rates (cyan). The idiosyncratic trend highlights two periods of anomaly, both associated with the zero lower bound (ZLB): the first after the Great Recession, and the second during the pandemic period. In the pandemic period, the idiosyncratic trend spikes to account for the nonlinearity of rates being stuck at the ZLB, preventing them from responding to the economic contraction. This trend reverses in the post-pandemic period, where negative values of the idiosyncratic trend reflect the slow response of the Federal Reserve to the shift in inflation expectations.

Now, let's consider the chart at the top, which shows the usual cyclical decomposition. Around the COVID period, the cyclical component of the federal funds rate is primarily explained by business cycle dynamics, with a minor role played by a positive spike in the idiosyncratic cycle, which helps the model account for the nonlinearity at the ZLB. As in the pre-pandemic period, the energy price cycle does not influence the dynamics of the rates. This may reflect the application of pre-pandemic parameters.

# 9 Conclusions

The link between slack in the economy and inflation over business cycles is not always apparent in unconditional correlations among variables. This is due to different types of demand and supply shocks affecting prices and real variables in heterogeneous ways, as well as structural transformations modifying long-run trends. We employed a multivariate unobserved components model informed by economic theory and survey data, capable of jointly estimating stable common cyclical components and time-varying trends, in order to uncover the cyclical relationships between real and nominal economic variables.

Our findings reveal a stable and significant negative correlation between inflation and economic slack, with consistent coefficients across more than a century of data, including various subsamples. This stability suggests that the relationship between demand policy and cyclical inflation developments has remained largely unchanged despite varying economic conditions.

The robust relationship we have identified is coherent with the view that policy has acted as a stabiliser for overall economic volatility. While the intensity of this moderating effect may have varied over time, the correlation between the output gap and inflation has remained consistent. While our approach doesn't provide a structural interpretation, the evidence strongly contradicts theories suggesting no correlation and highlights the importance of considering economic slack in inflation dynamics.

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Online Appendix to

# A Hundred Years of Business Cycles and the Phillips curve

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October 1, 2024

**Abstract**

This Online Appendix provides the estimation procedure and all the charts for each the models described in 'A Hundred Years of Business Cycles and the Phillips curve'.

**JEL classification:** E31, E32.

**Keywords:** Phillips Curve, Semi-structural models.

# Contents

# A Additional results

## A.1 Baseline model, sample 1900-2019

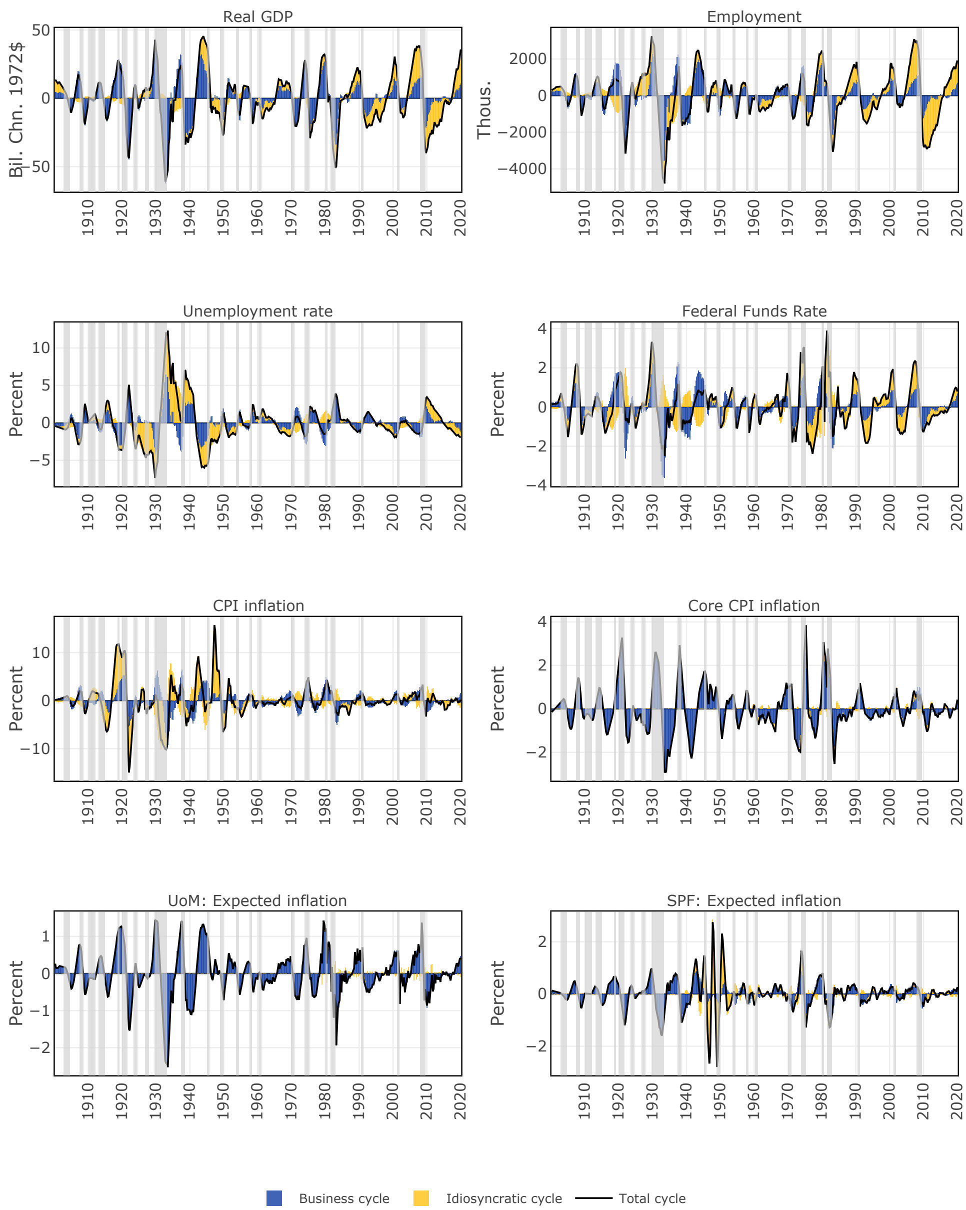



Figure 1: Historical decomposition of the cycles, as estimated by the model. The chart reports the business cycle (in blue), and idiosyncratic cycle (in yellow). The model is estimated over the sample 1900-2019.

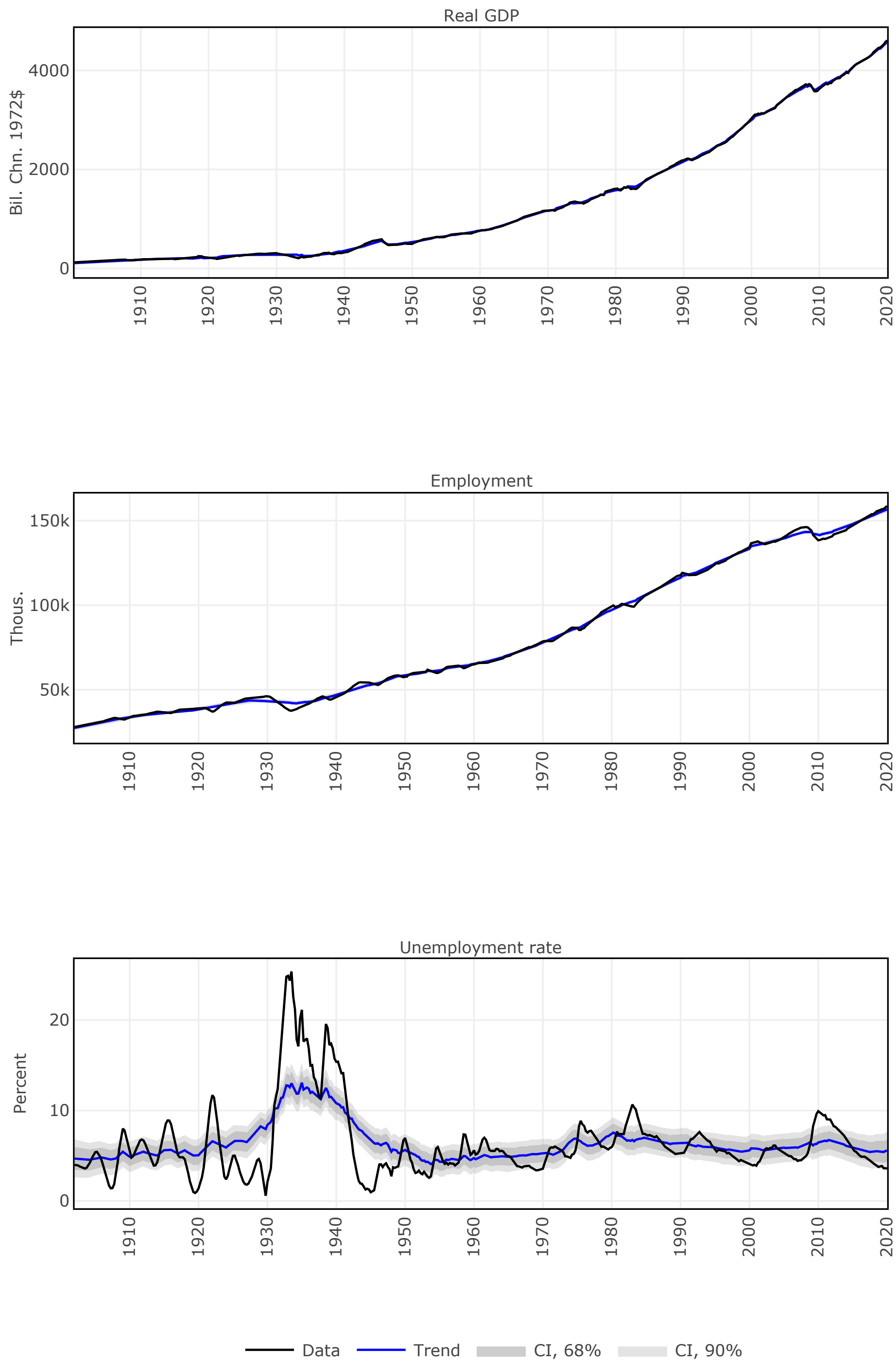



Figure 2: Independent trends of output, employment, unemployment, and oil prices (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model. The model is estimated over the sample 1900-2019.

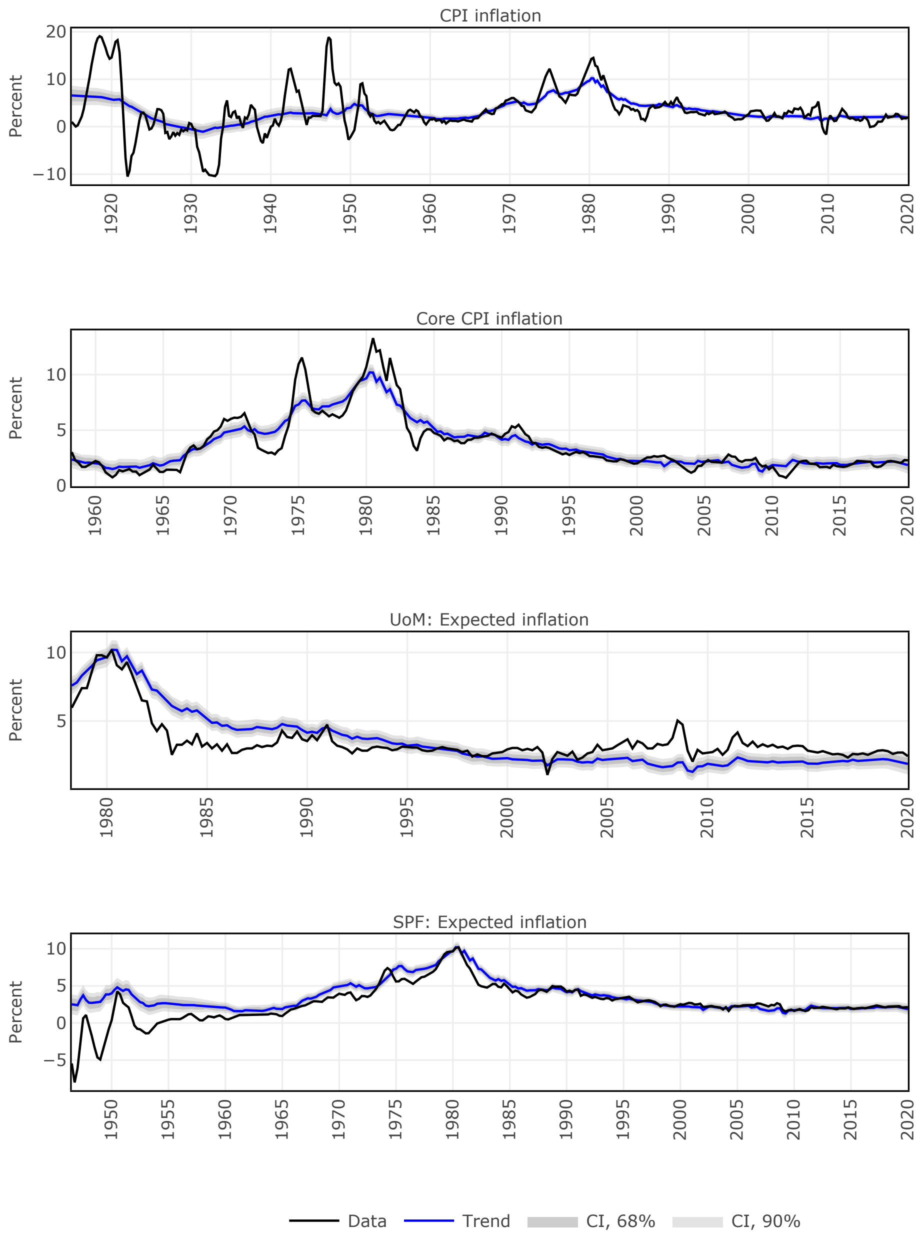



Figure 3: Trend common to CPI inflation, core CPI inflation, and inflation expectations (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model.The model is estimated over the sample 1900-2019.

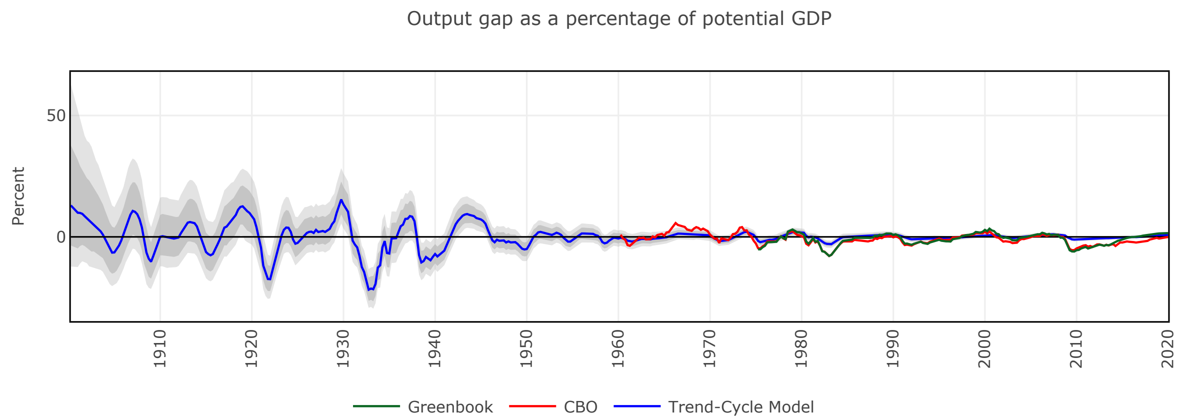



Figure 4: Output gap

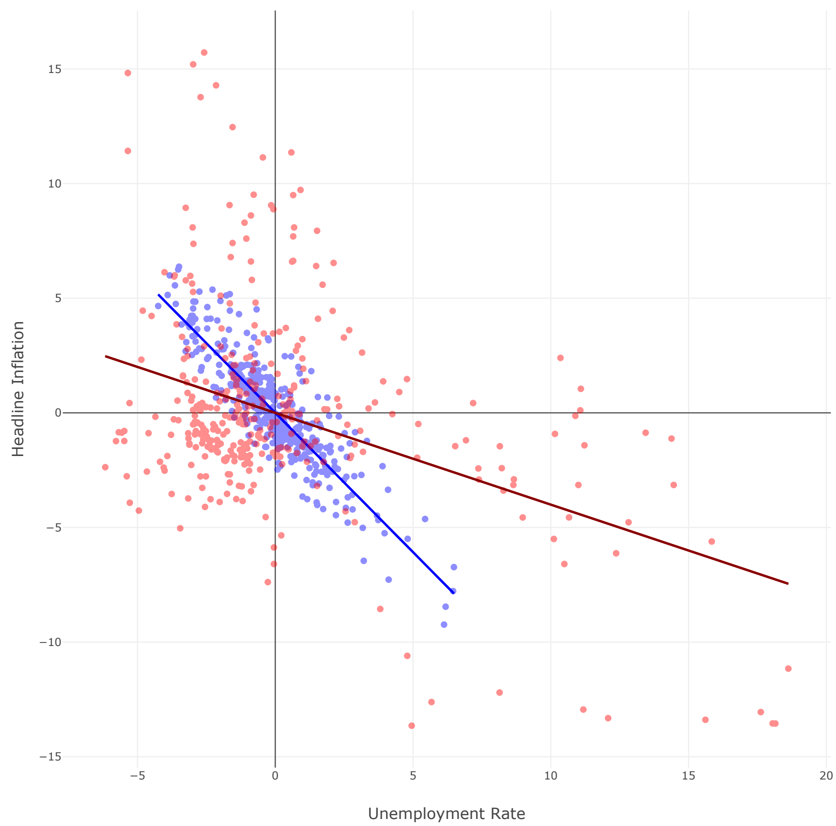
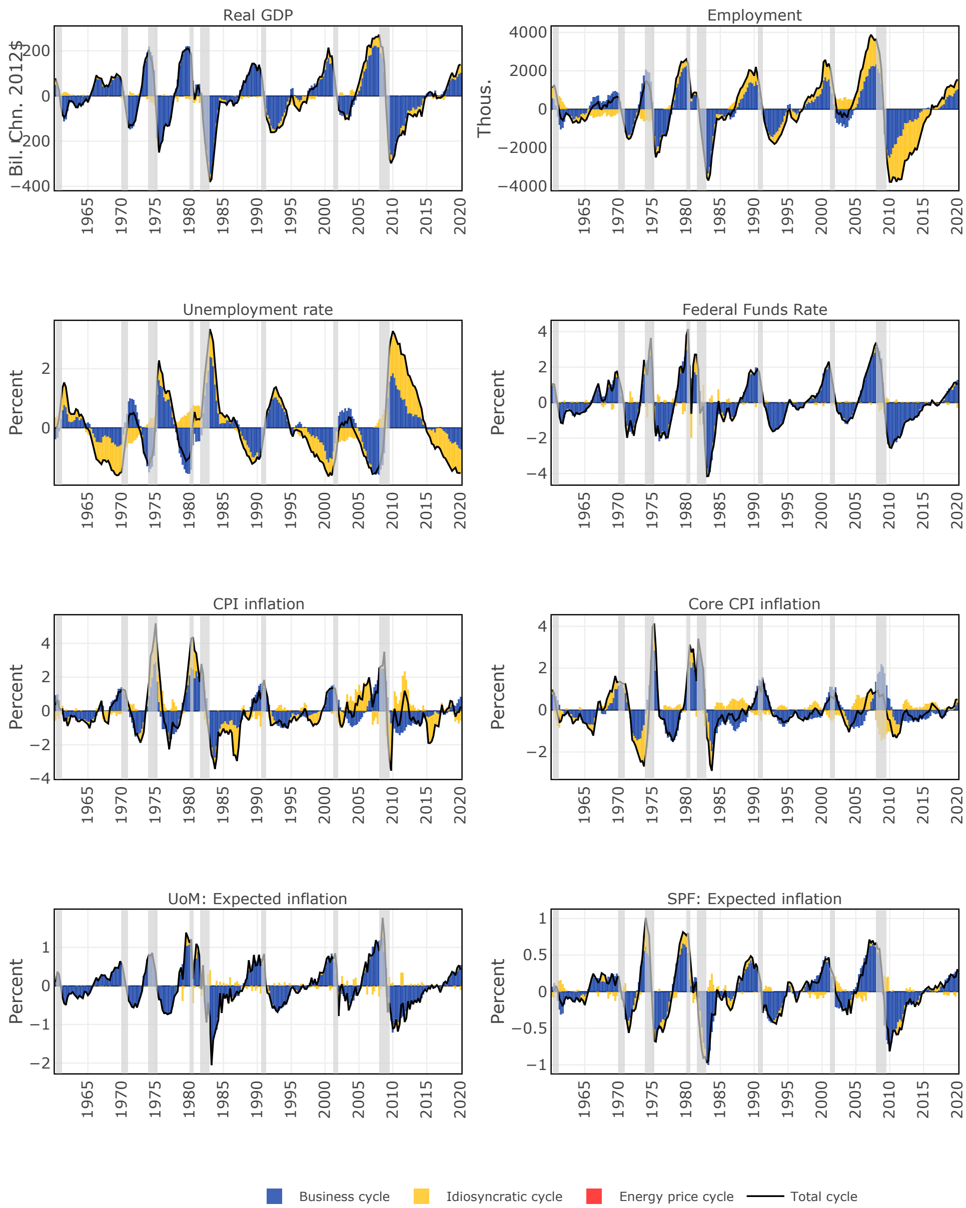



Figure 5: This chart plots the business cycle component of CPI inflation against the business cycle component of the unemployment rate (blue dots) and the corresponding bivariate linear regression line (blue line). The chart also plots demeaned CPI inflation against the demeaned unemployment rate (red dots) and the corresponding bivariate linear regression line (red line).

## A.2 Baseline model, sample 1960/2019

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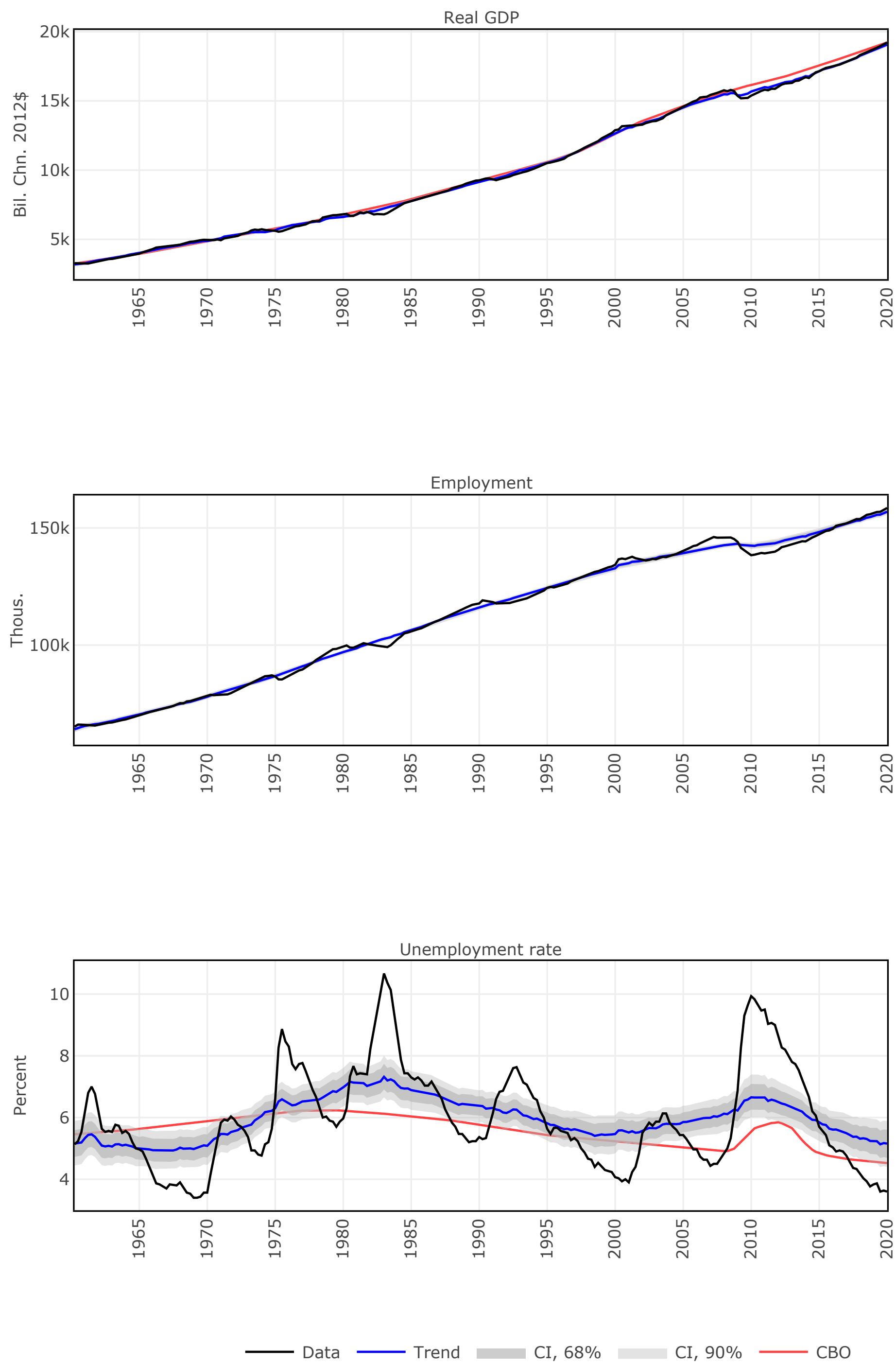


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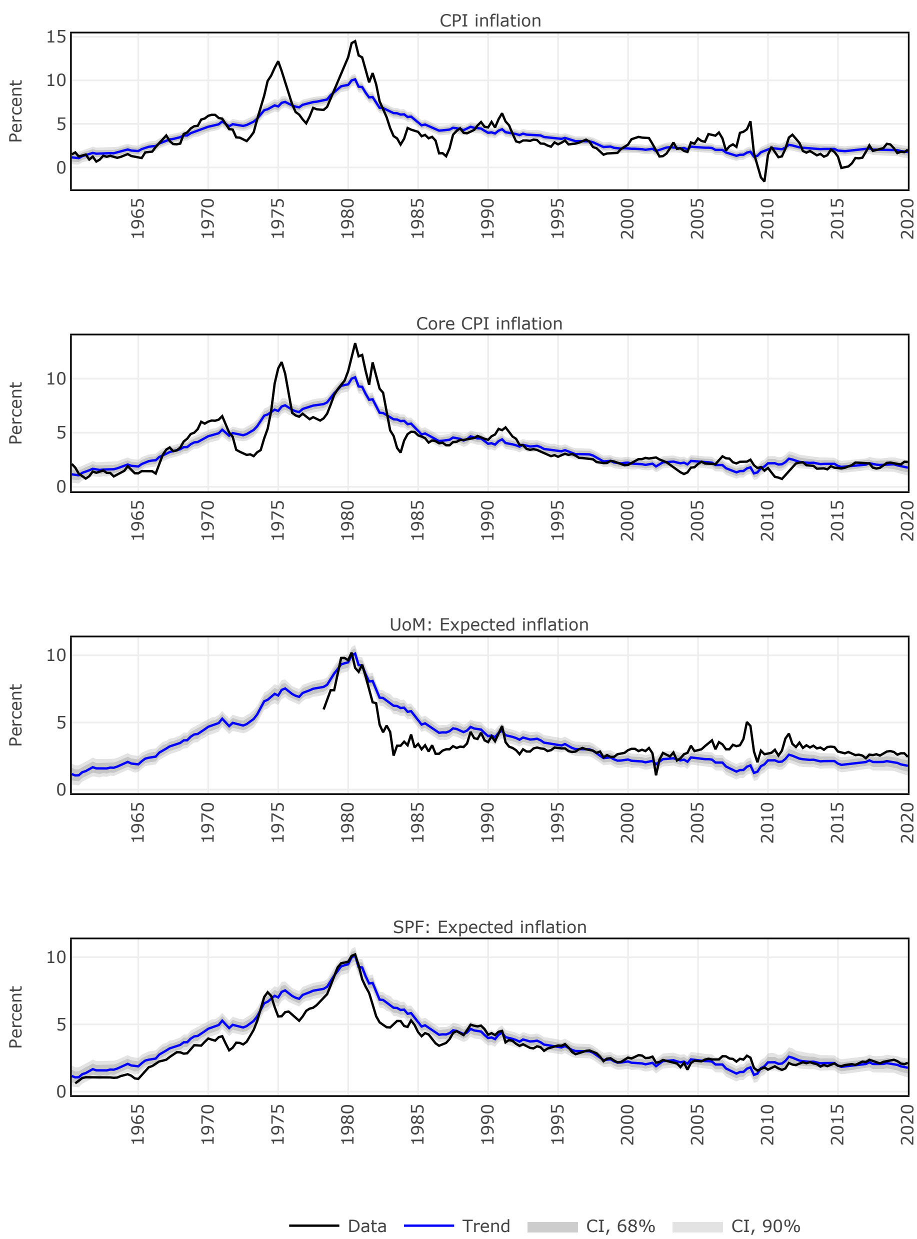


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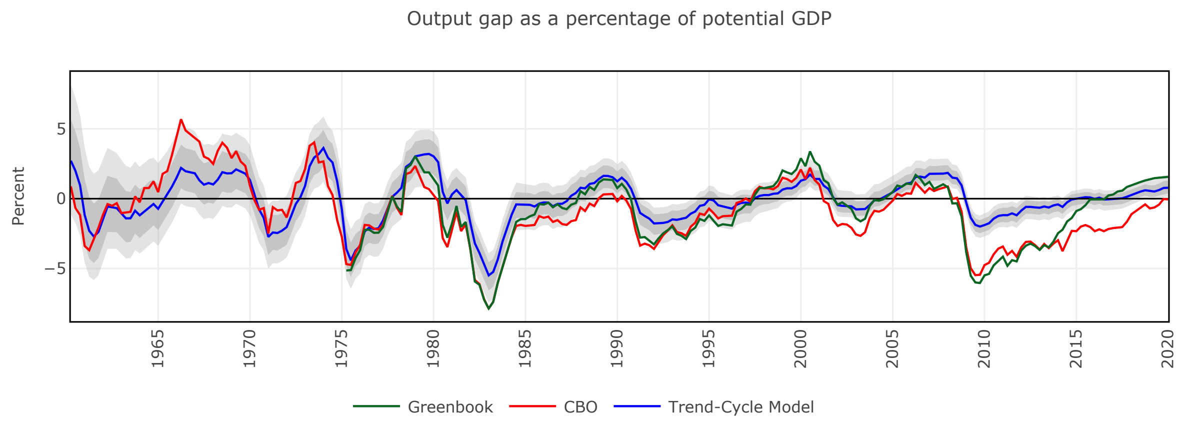
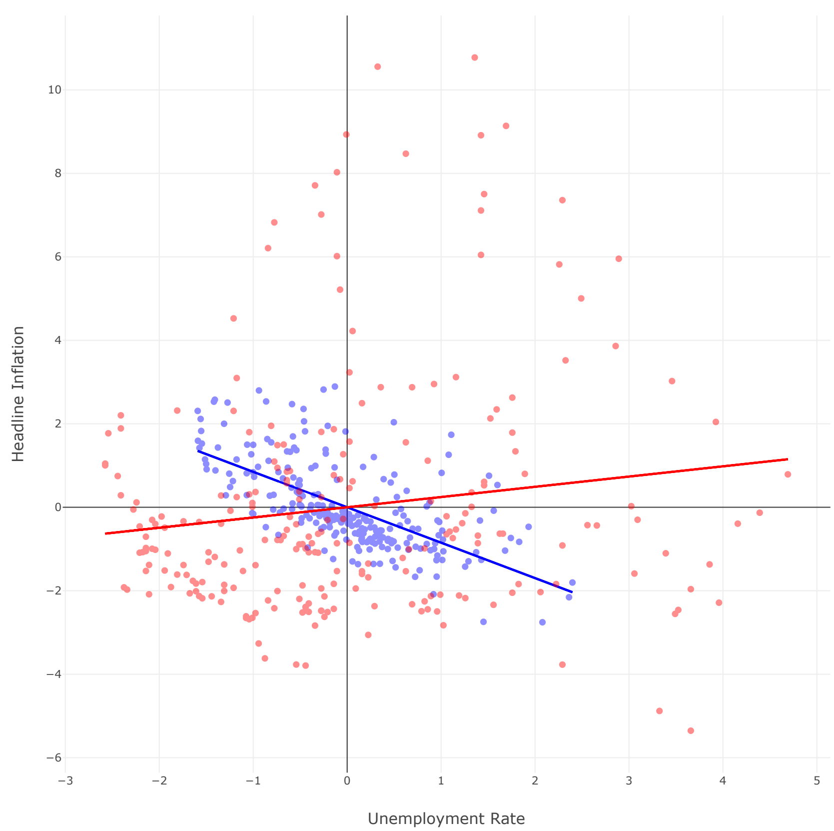



Figure 9: Output gap

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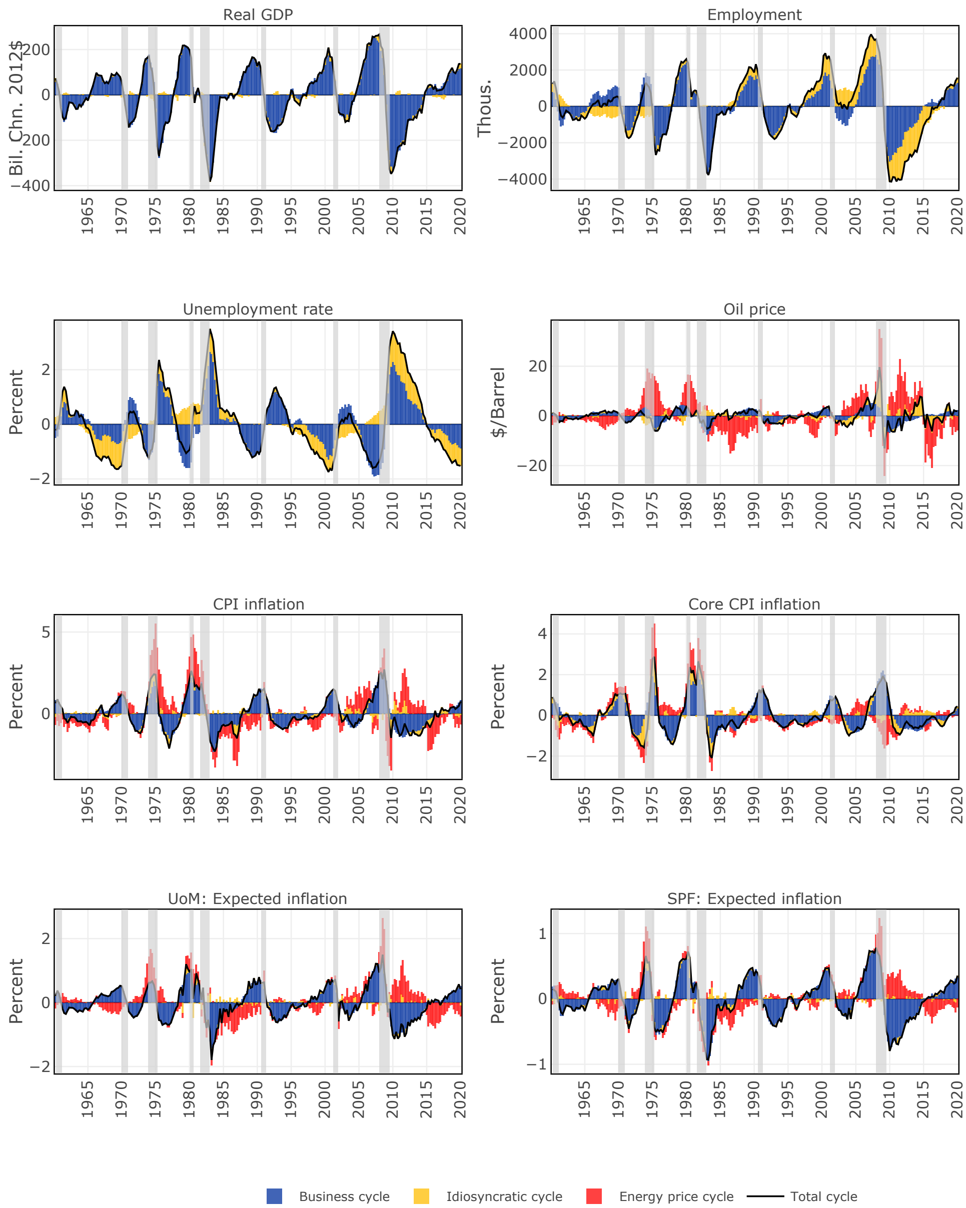
## A.3 Model with oil prices, FFR not responding to oil, sample 1960-2019

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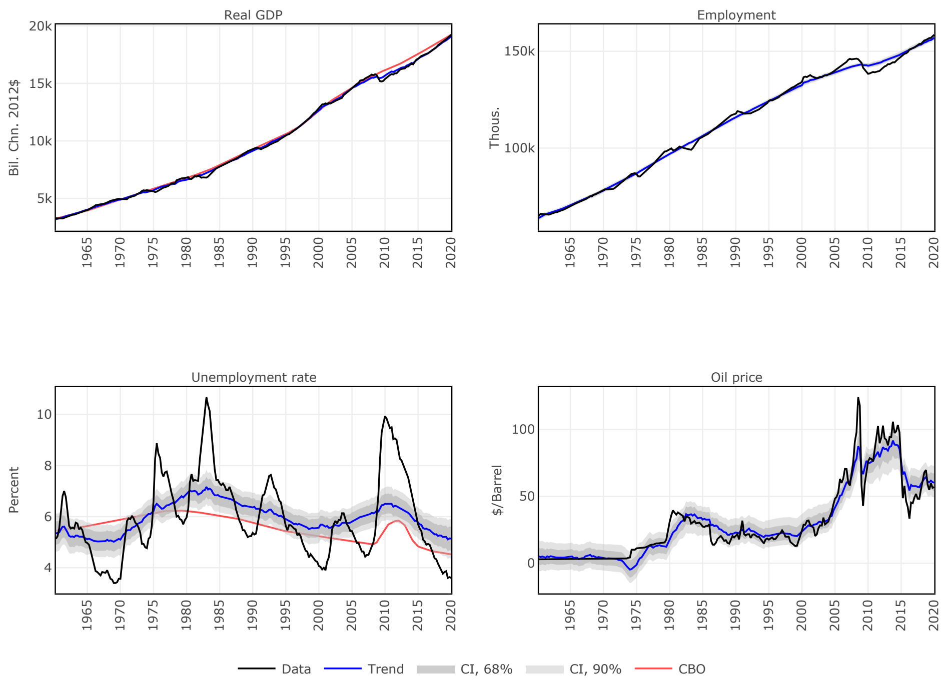


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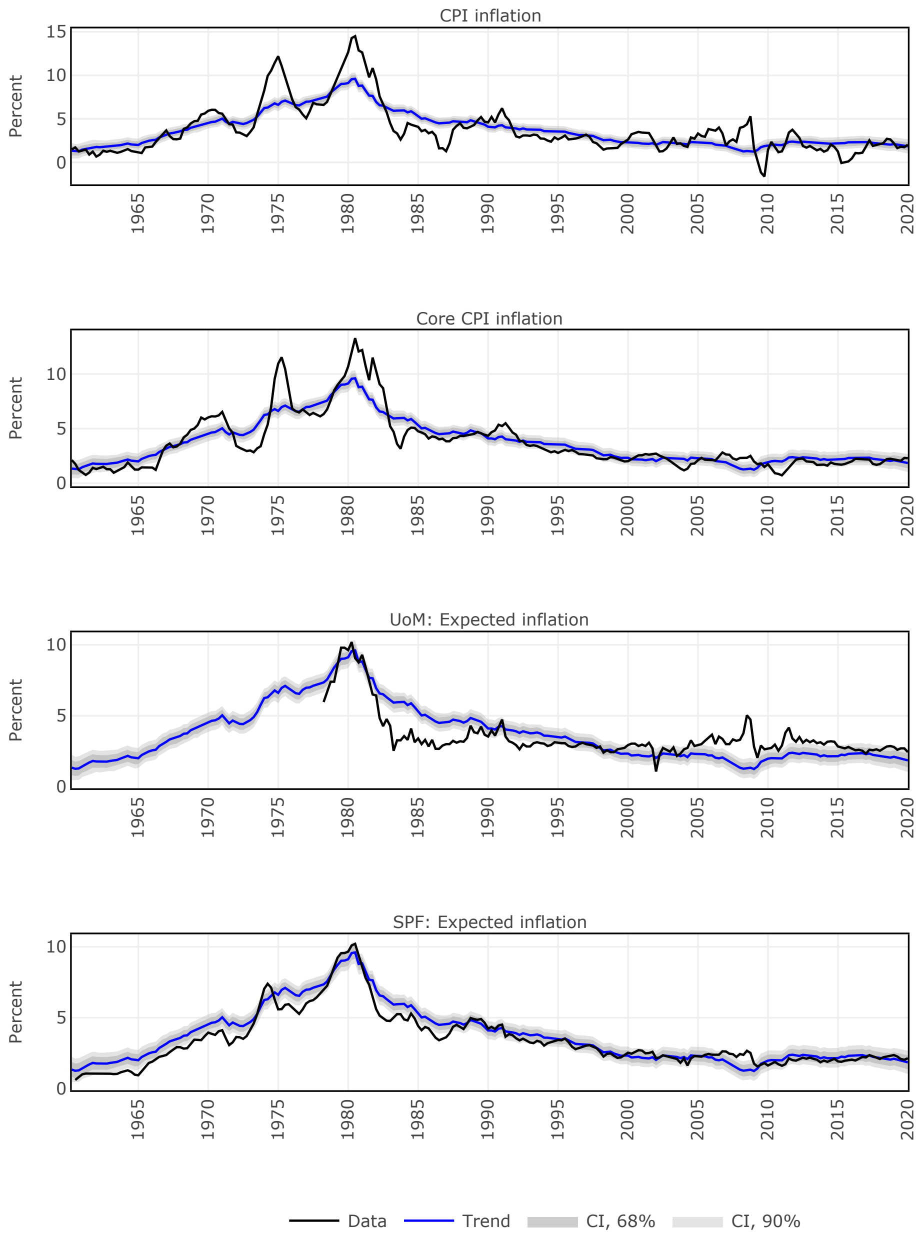


Figure 13: Trend common to CPI inflation, core CPI inflation, and inflation expectations (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model.The model is estimated over the sample 1960-2019.

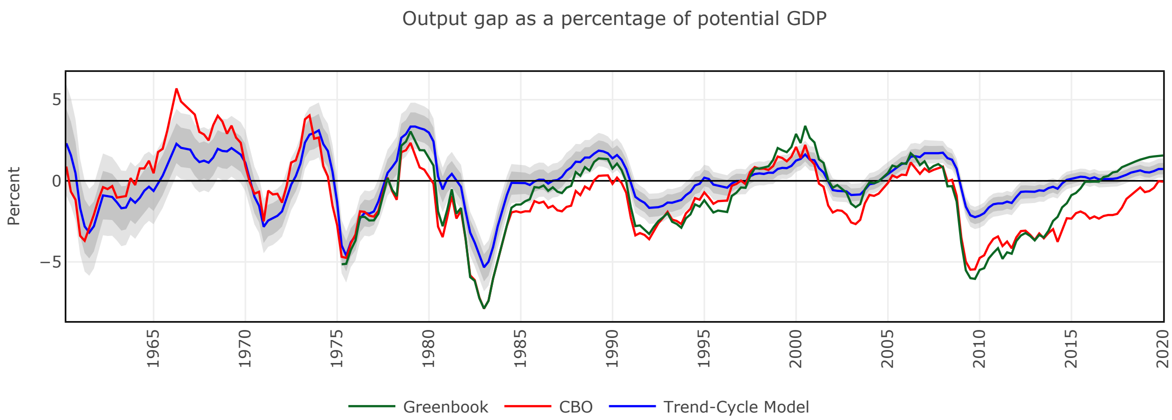


Figure 14: Output gap

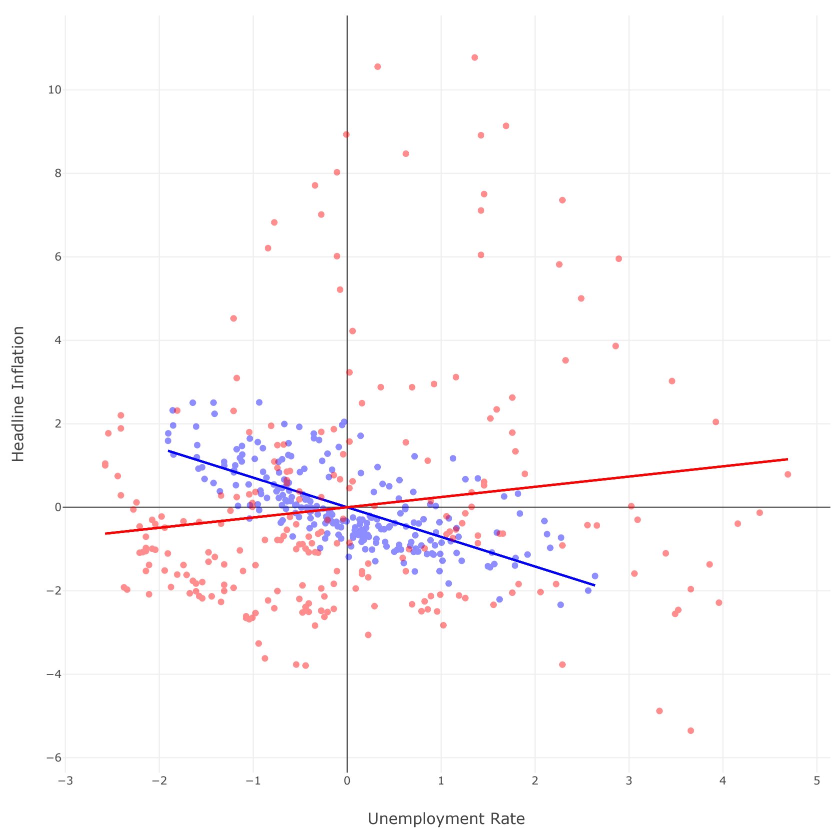


Figure 15: This chart plots the business cycle component of CPI inflation against the business cycle component of the unemployment rate (blue dots) and the corresponding bivariate linear regression line (blue line). The chart also plots demeaned CPI inflation against the demeaned unemployment rate (red dots) and the corresponding bivariate linear regression line (red line).

## A.4 Model with oil prices, FFR not responding to oil, sample 1960-2019

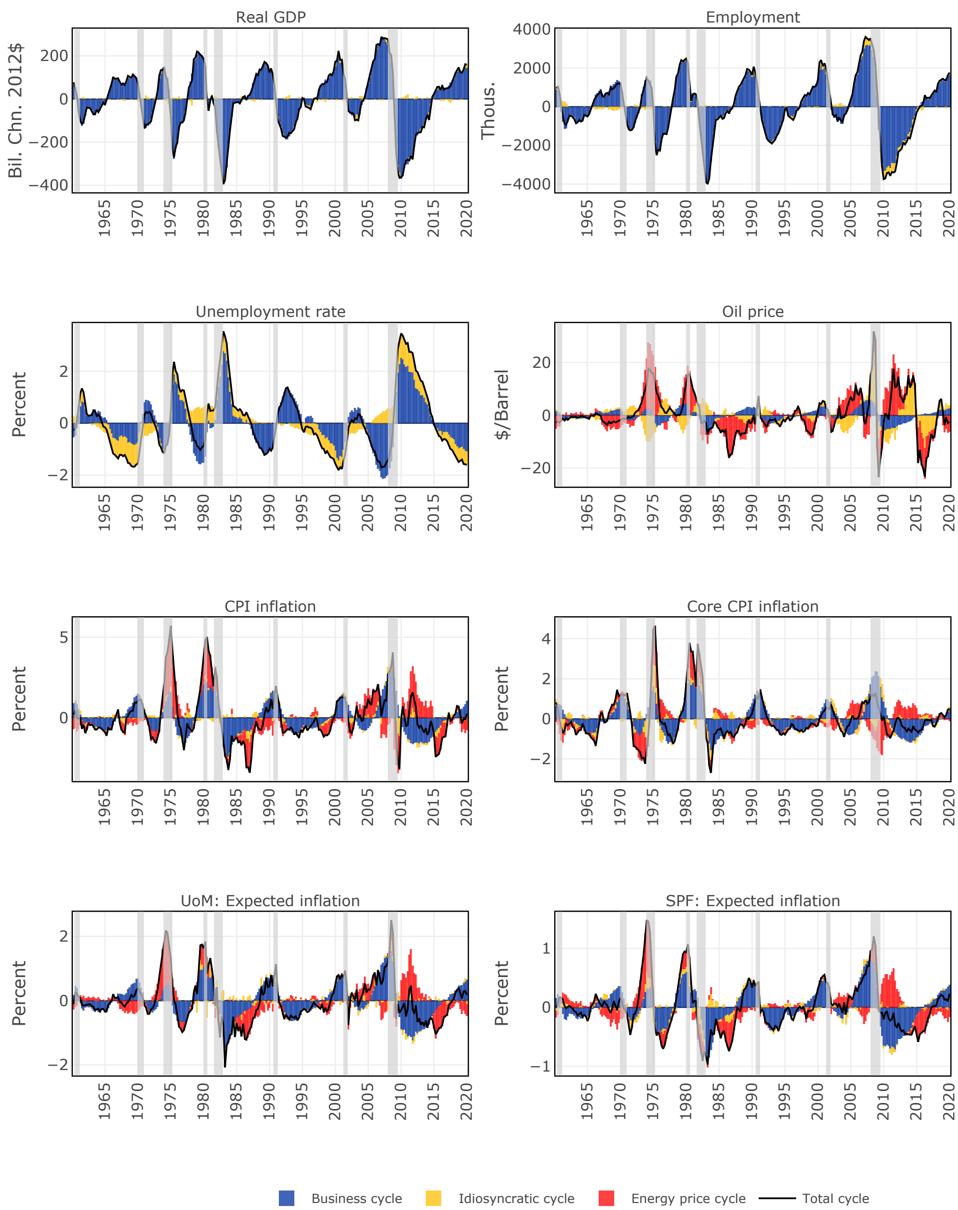


Figure 16: Historical decomposition of the cycles, as estimated by the model. The chart reports the business cycle (in blue), Energy price cycle (in red),and idiosyncratic cycle (in yellow). The model is estimated over the sample 1960-2019.

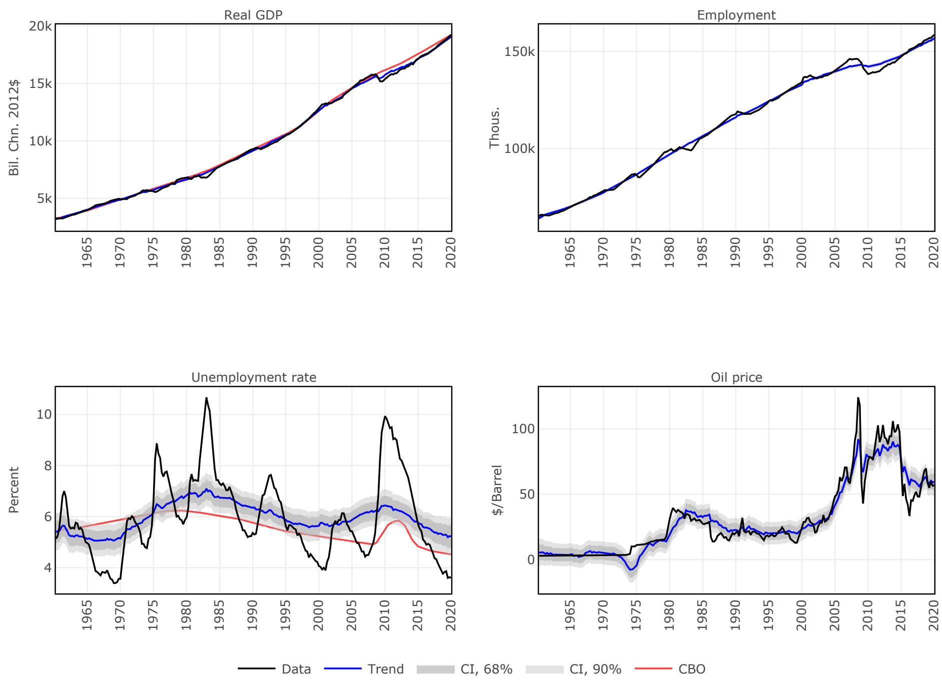


Figure 17: Independent trends of output, employment, unemployment, and oil prices (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model. The model is estimated over the sample 1960-2019.

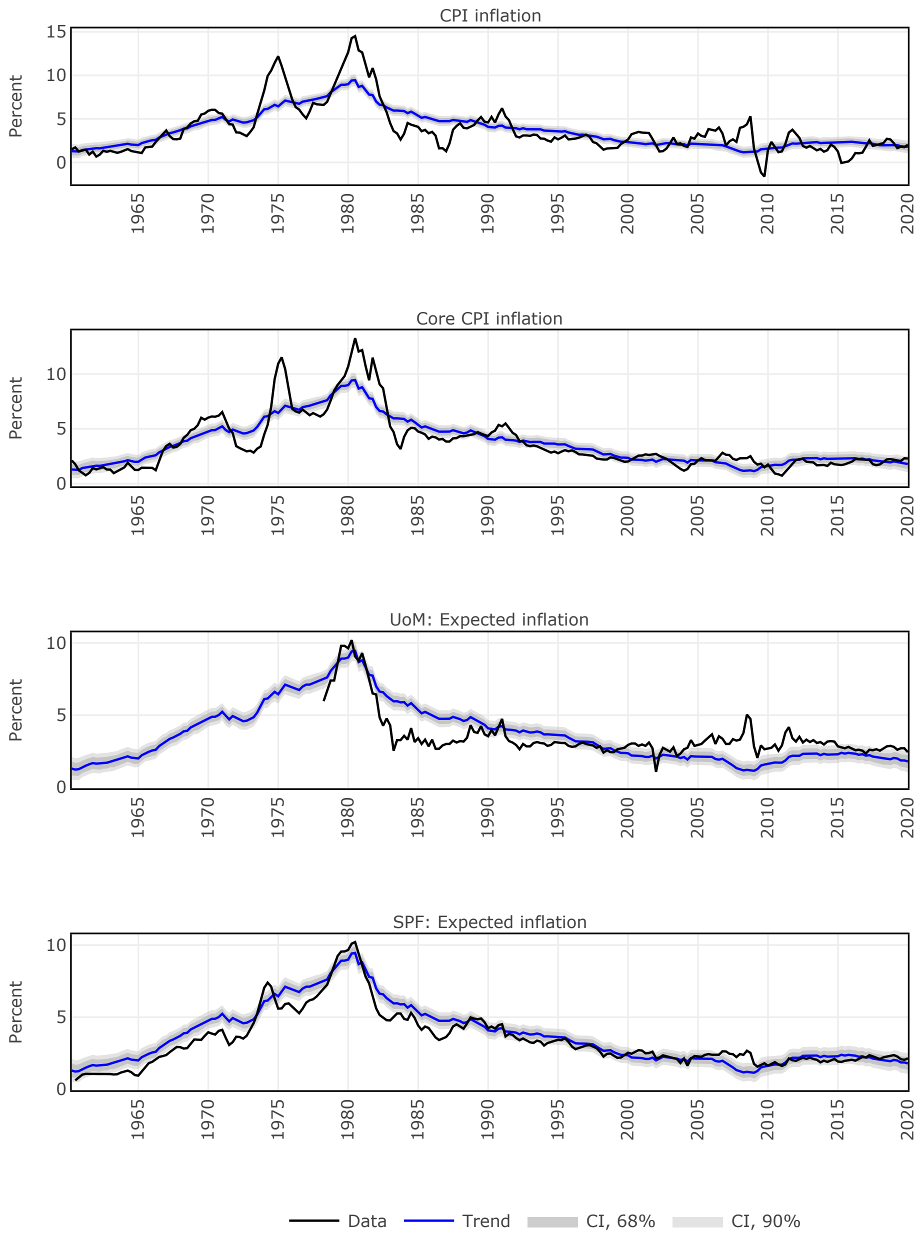


Figure 18: Trend common to CPI inflation, core CPI inflation, and inflation expectations (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model.The model is estimated over the sample 1960-2019.

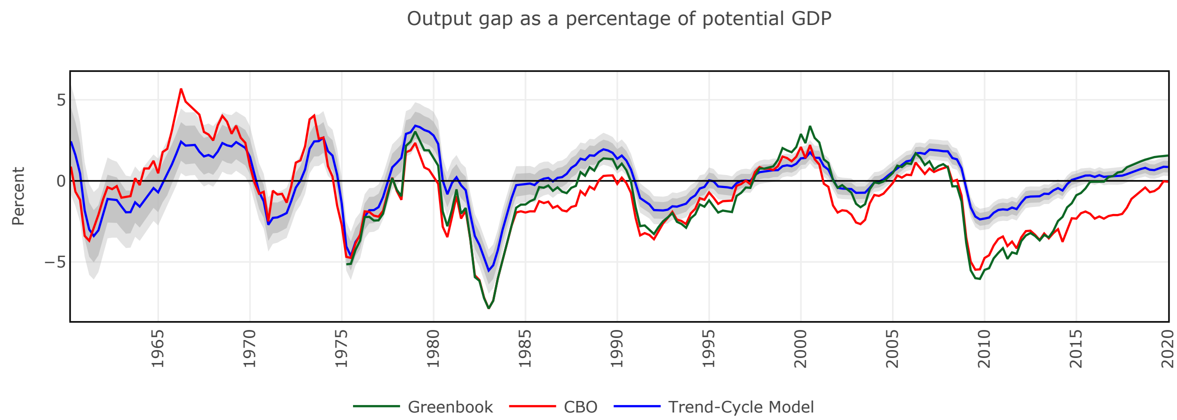


Figure 19: Output gap

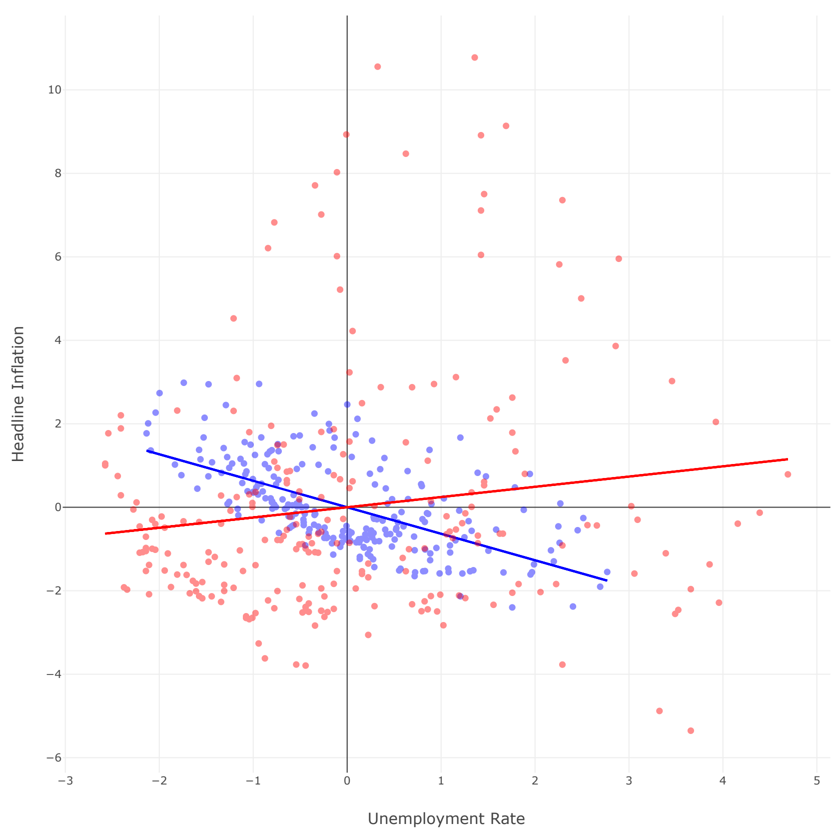


Figure 20: This chart plots the business cycle component of CPI inflation against the business cycle component of the unemployment rate (blue dots) and the corresponding bivariate linear regression line (blue line). The chart also plots demeaned CPI inflation against the demeaned unemployment rate (red dots) and the corresponding bivariate linear regression line (red line).

## A.5 The extended COVID sample

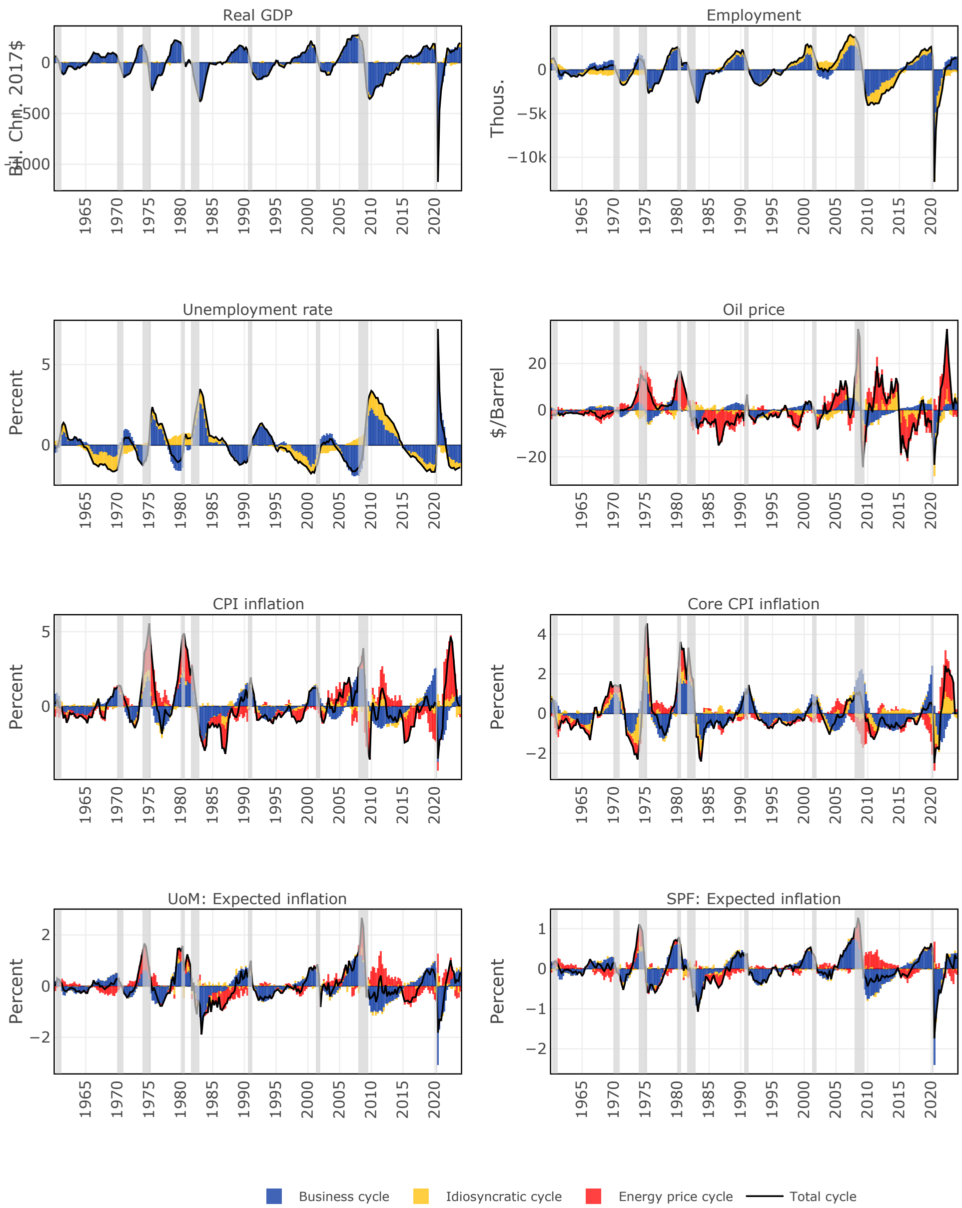


Figure 21: Historical decomposition of the cycles, as estimated by the model. The chart reports the business cycle (in blue), and idiosyncratic cycle (in yellow). The model is estimated over the sample 1960-2019.

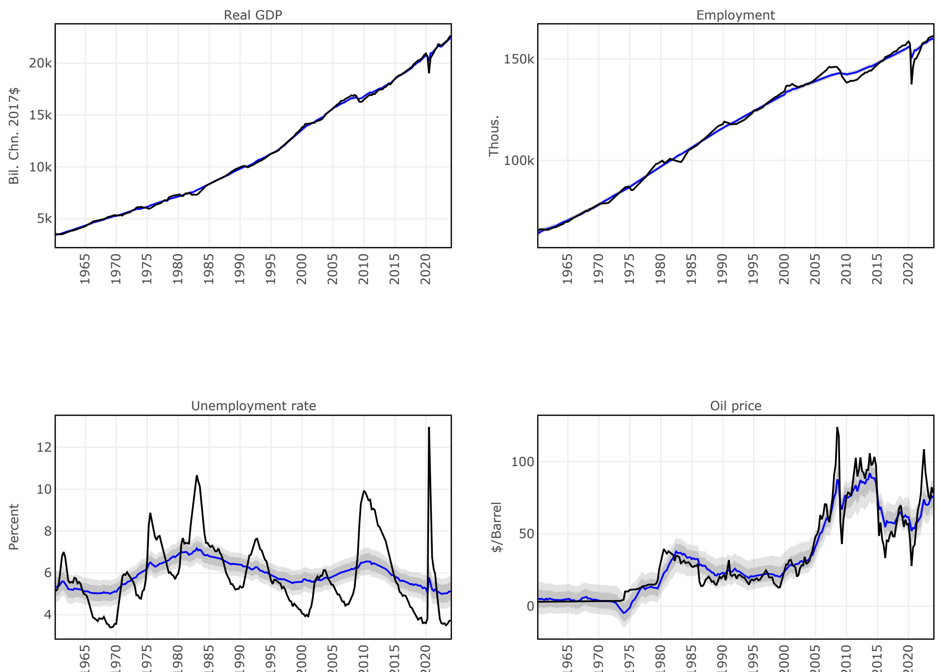


Figure 22: Independent trends of output, employment, unemployment, and oil prices (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model. The model is estimated over the sample 1960-2019.

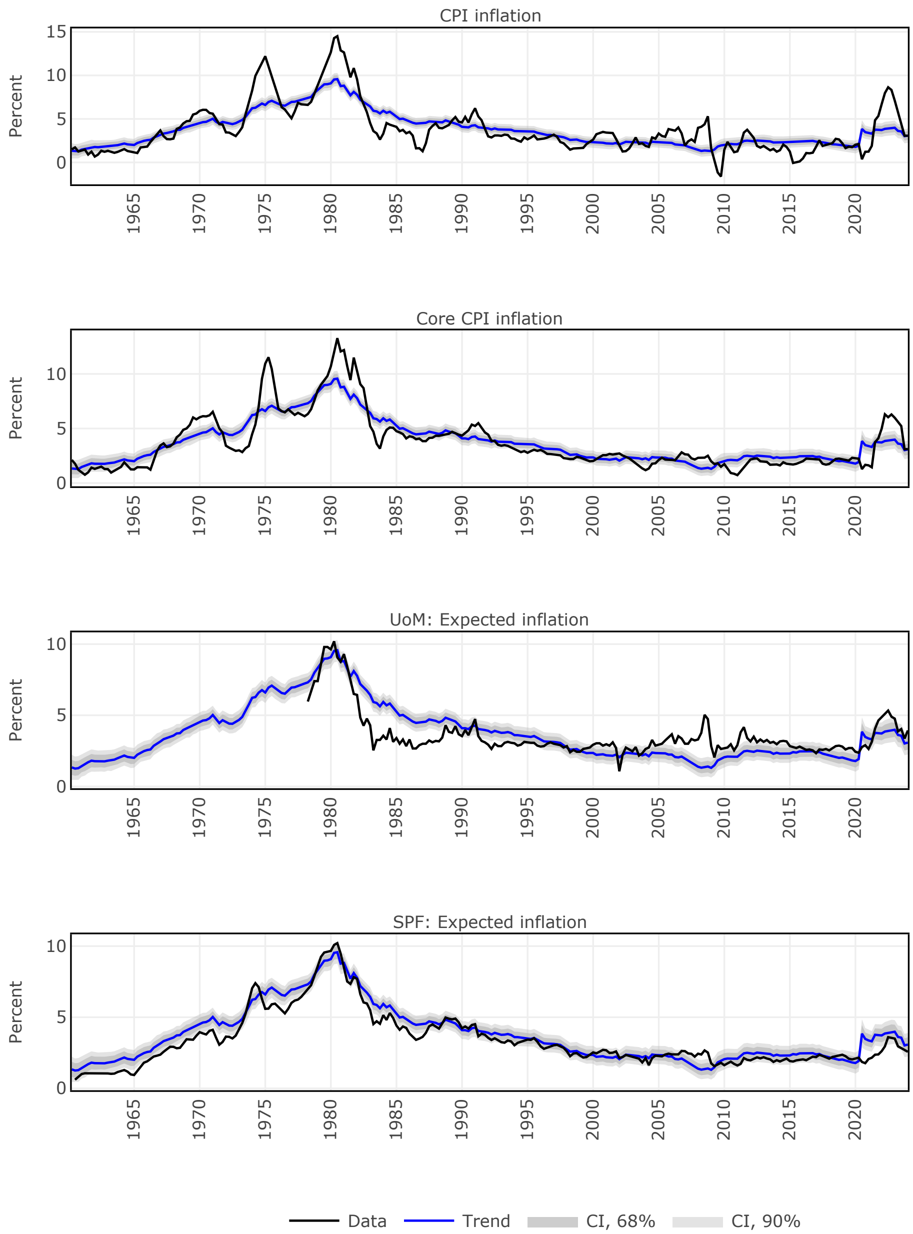


Figure 23: Trend common to CPI inflation, core CPI inflation, and inflation expectations (in blue), with coverage intervals at 68% coverage (dark shade) and 90% coverage (light shade), as estimated by the model.The model is estimated over the sample 1960-2019.

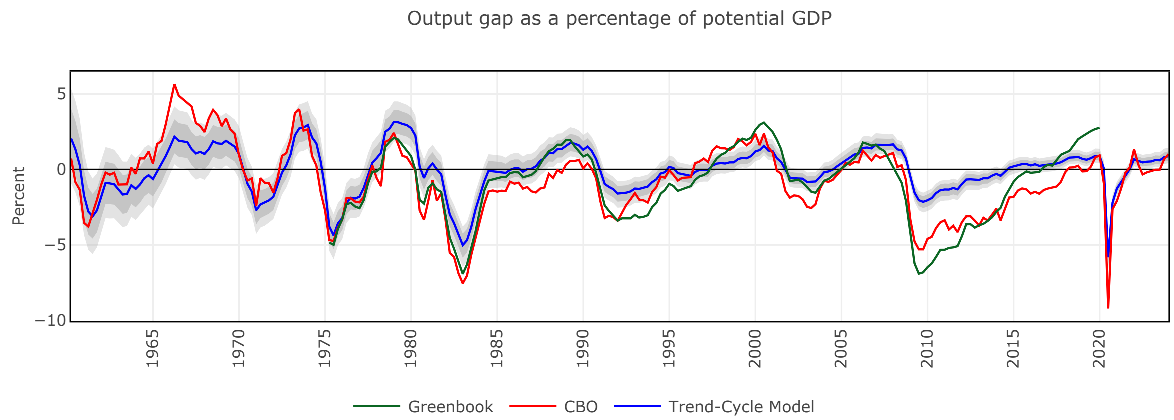


Figure 24: Output gap

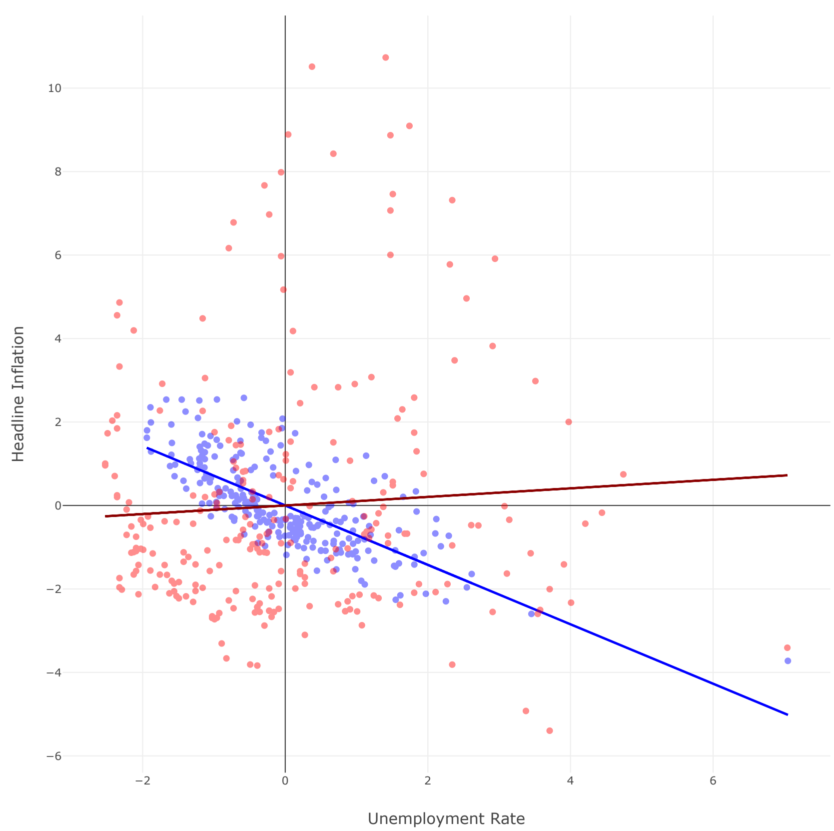


Figure 25: This chart plots the business cycle component of CPI inflation against the business cycle component of the unemployment rate (blue dots) and the corresponding bivariate linear regression line (blue line). The chart also plots demeaned CPI inflation against the demeaned unemployment rate (red dots) and the corresponding bivariate linear regression line (red line).

# B Stability of the model

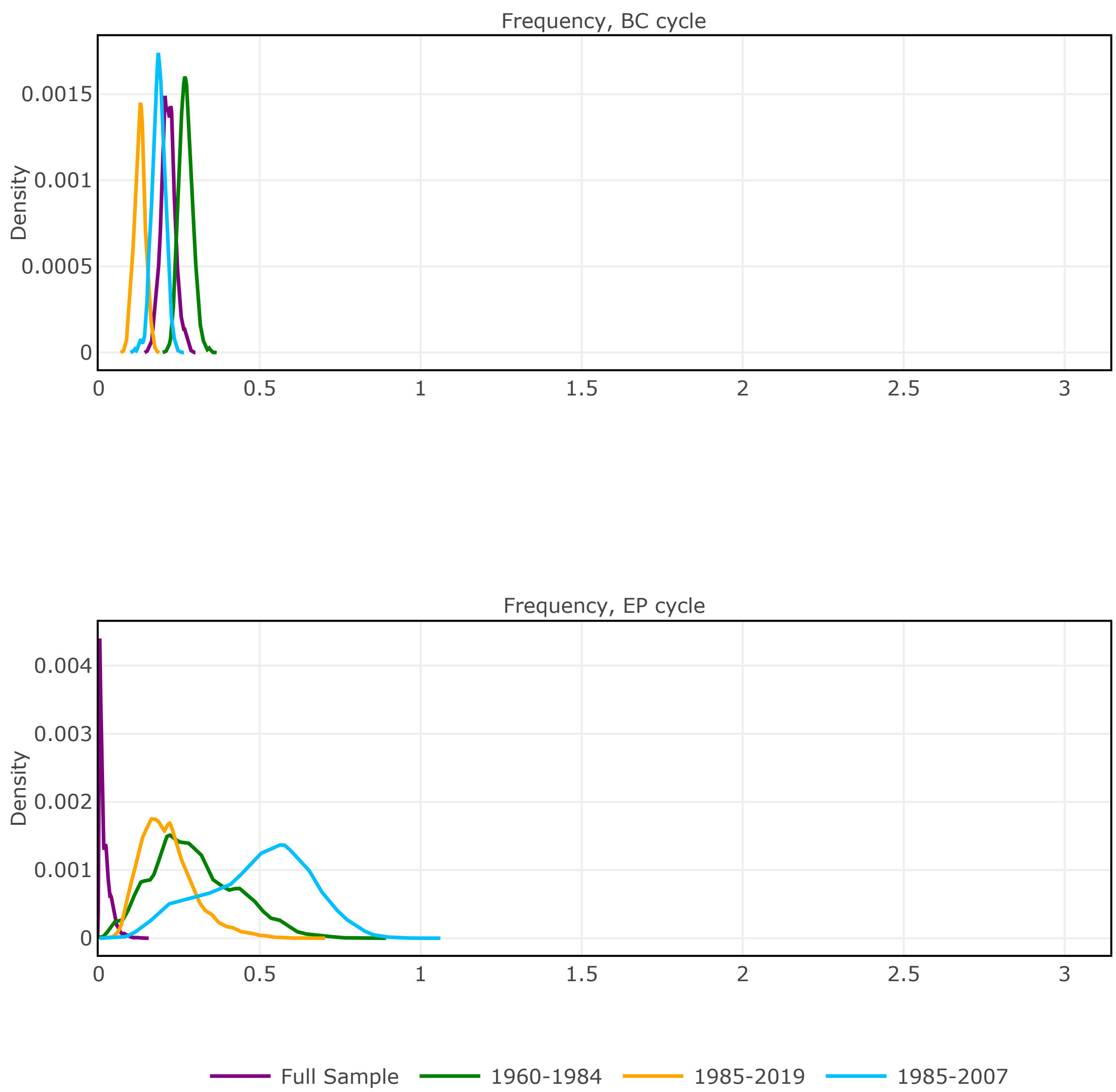


Figure 26: The chart shows the frequency of the business cycle (top) and the energy price cycle (bottom) for the samples 1960-2019, 1960-1984, 1985-2019, and 1985-2007.

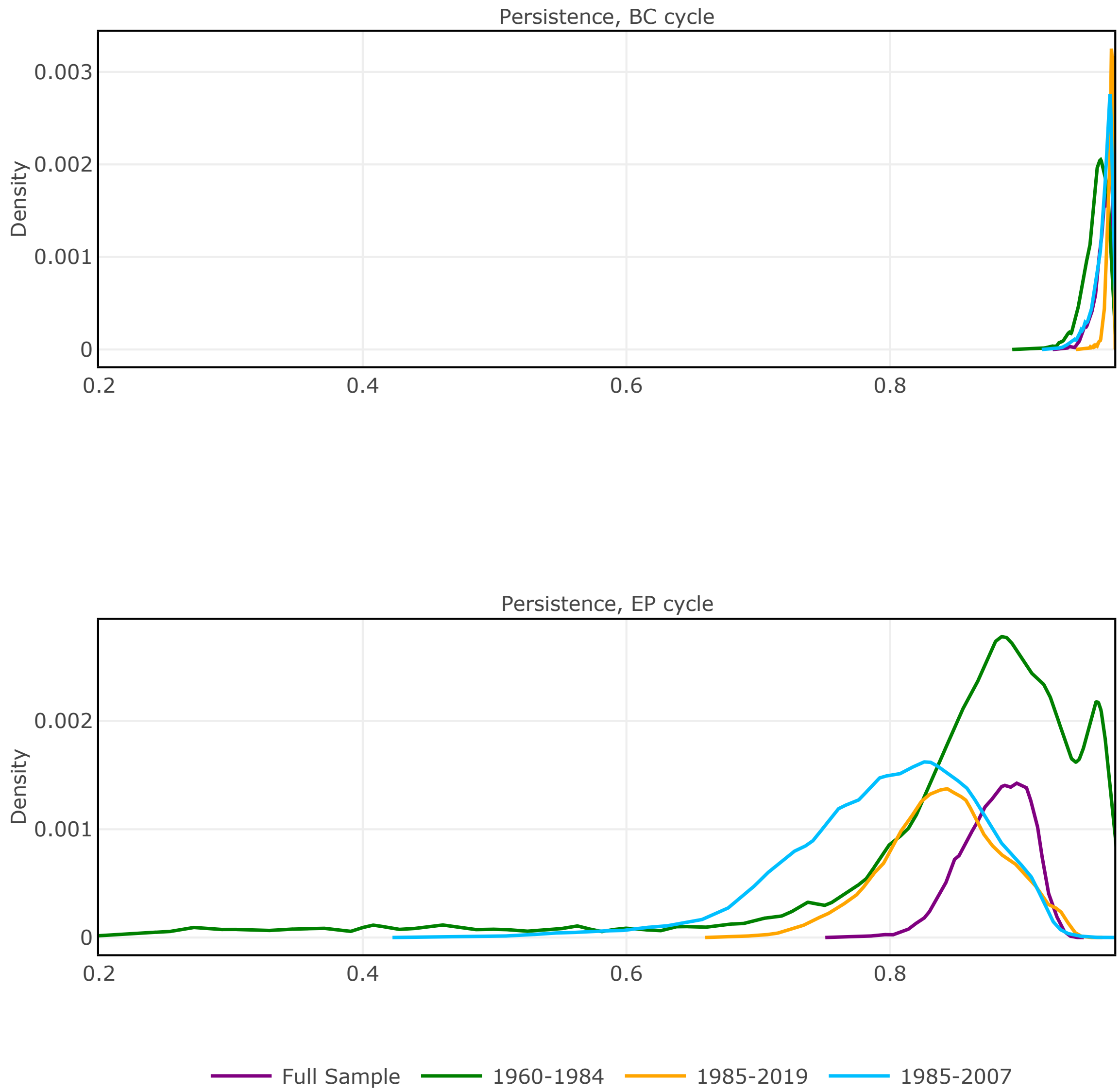


Figure 27: The chart shows the persistence of the business cycle (top) and the energy price cycle (bottom) for the samples 1960-2019, 1960-1984, 1985-2019, and 1985-2007.

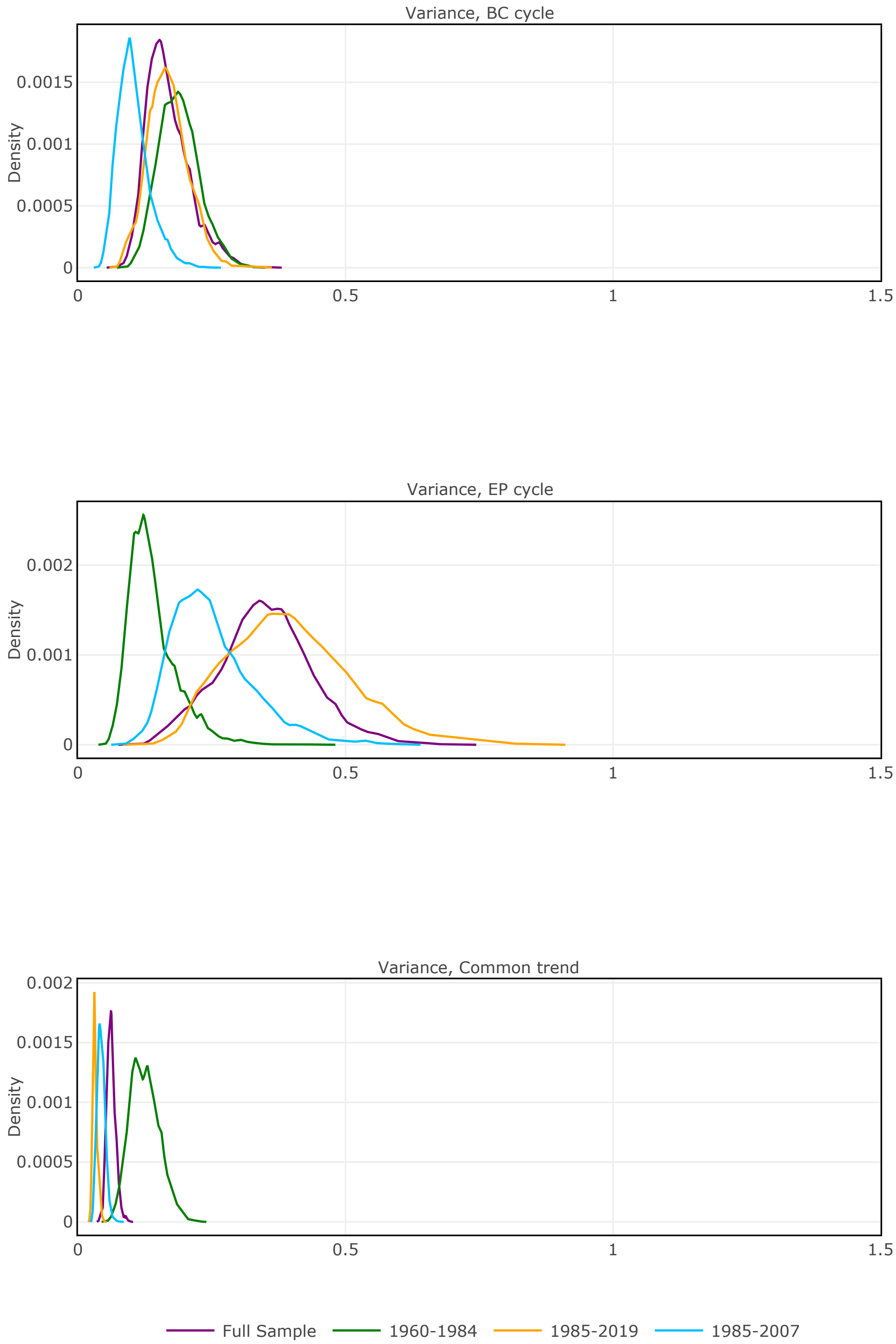


Figure 28: The chart shows the variance of the business cycle (top), the energy price cycle (middle) and the common trend of the nominal variables (bottom) for the samples 1960-2019, 1960-1984, 1985-2019, and 1985-2007.

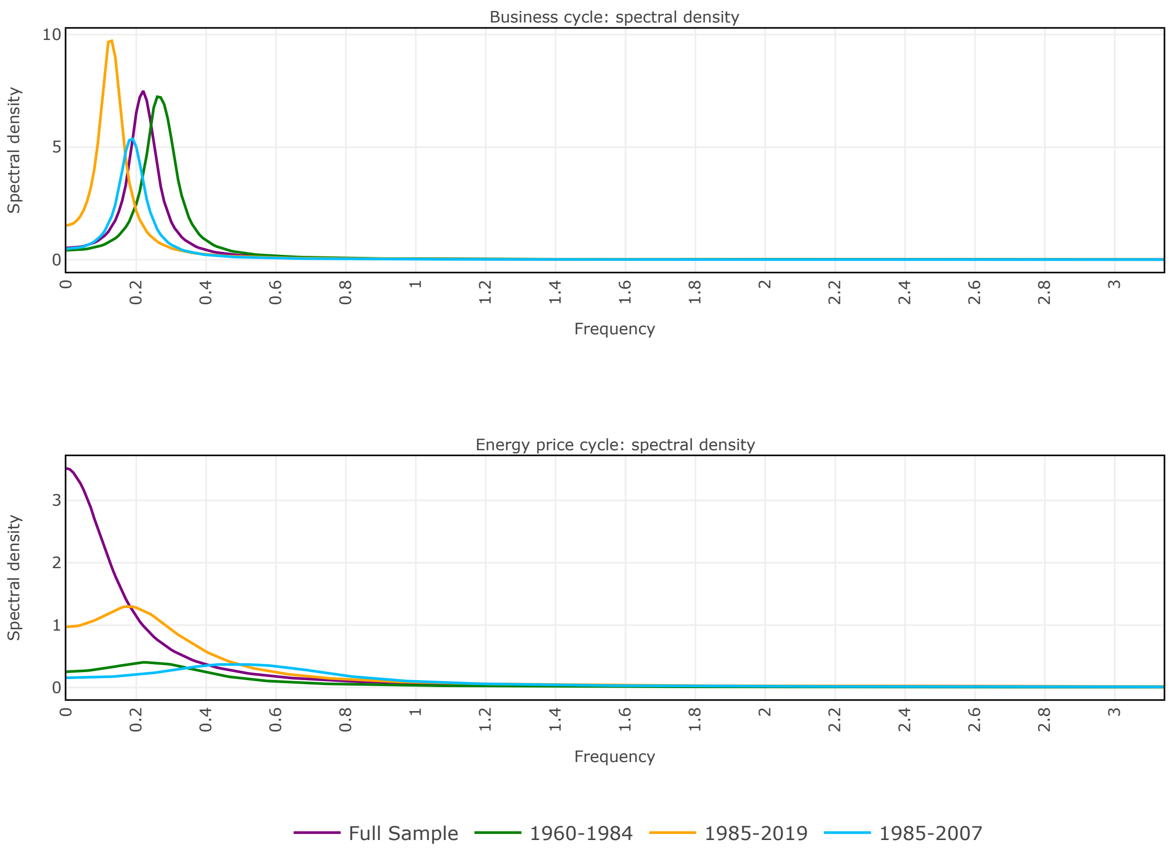


Figure 29: The chart shows the spectral density of the business cycle (top) and the energy price cycle (bottom) for the samples 1960-2019, 1960-1984, 1985-2019, and 1985-2007.

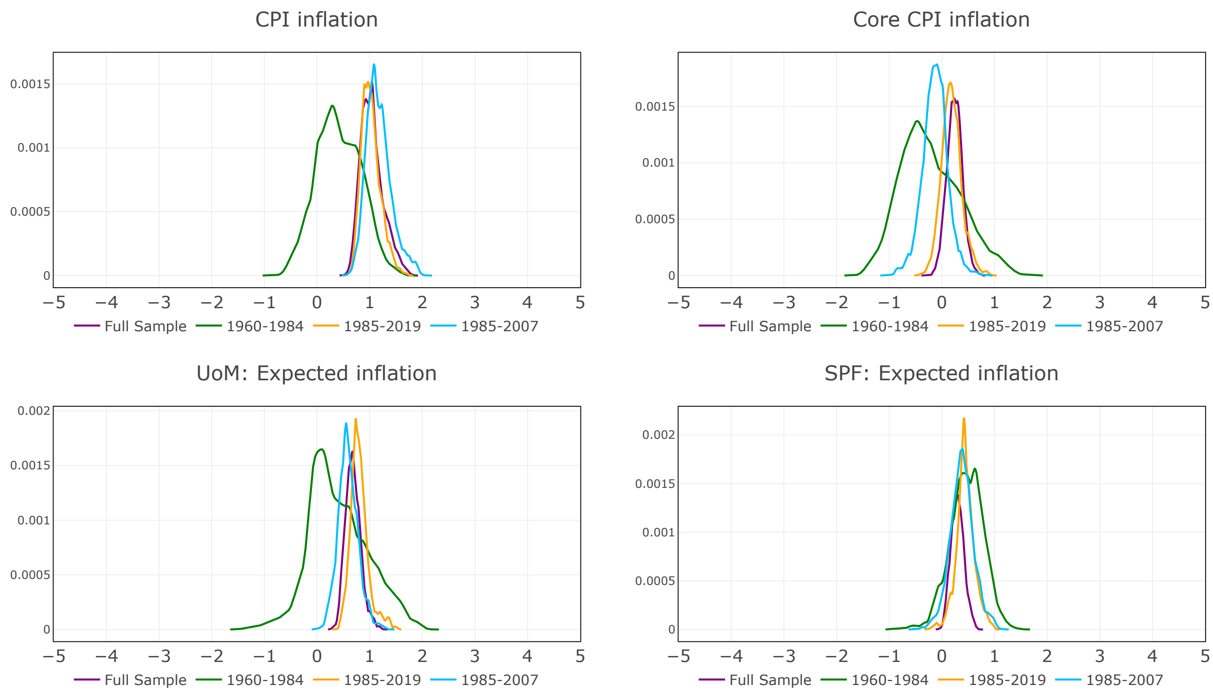


Figure 30: The chart shows the posterior distributions for the coefficients of the contemporaneous response of individual variables to energy price cycle for for the samples 1960-2019, 1960-1984, 1985-2019, and 1985-2007.

# C Rolling windows

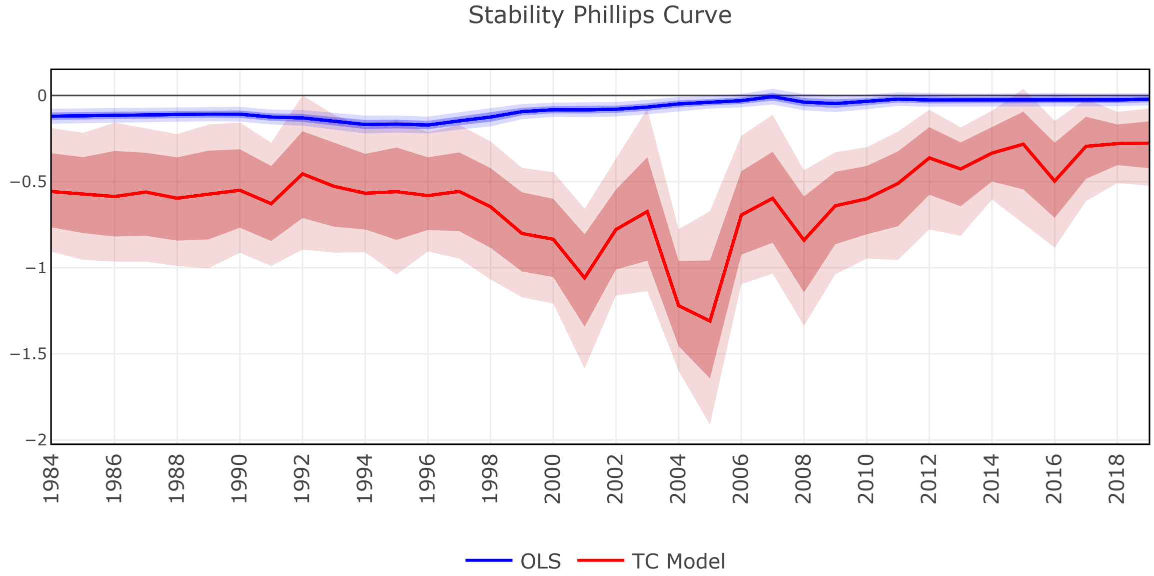


Figure 31: Stability Phillips Curve

# D Adaptive Metropolis-Within-Gibbs

## D.1 Algorithm

The estimation algorithm is an improved version of the Metropolis-Within-Gibbs in Hasenzagl et al. (2022) that employs the Single Component Adaptive Metropolis proposed in Haario et al. (2005).

This hybrid algorithm is structured in two blocks: (1) a Single Component Adaptive Metropolis (Haario et al., 2005) step for the estimation of the state-space parameters, (2) a Gibbs sampler (Koopman and Durbin, 2000; Jarociński, 2015) to draw the unobserved states conditional on the model parameters. Since we have non-stationary unobserved states, we use the Kalman filter with exact diffuse initial conditions (Koopman and Durbin, 2000; Durbin and Koopman, 2012) to compute the log-likelihood of the model. Finally, we used the priors in Hasenzagl et al. (2022).

**Algorithm: Adaptive Metropolis-Within-Gibbs**

- Initialisation

  Let $\mathscr{K} := \{1, \dots, n_k\}$ and denote as $\mathbf{P}(\mathscr{K})$ a function that returns a random permutation of $\mathscr{K}$ (uniformly taken from the full set of permutations of $\mathscr{K}$). Let also $\boldsymbol{\theta}_0$ be a $n_k$ dimensional vector corresponding to the initial value for the Metropolis parameters. This vector is associated to a high posterior mass.

- Single component adaptive metropolis

  **let** $m = 1$

  **for** $j = 1, \dots, 10000$

  **let** $\mathbf{S}_j = \mathbf{P}(\mathscr{K})$

**for each** $k$ in $\mathbf{S}_j$

1. *Adaptation:* Update the standard deviation of the proposal distribution

$$\sigma_{k,j} = \begin{cases} 1 & \text{if } j \leq 10, \\ \exp\big(\alpha_{k,j-1} - 0.44\big)\sigma_{k,j-1} & \text{otherwise,} \end{cases}$$

   where $\alpha_{k,j-1}$ is the acceptance rate for the iteration $j-1$, for the parameter at position $S_{k,j}$. Besides, 44% is the standard target acceptance rate for single component Metropolis algorithms.

2. *New candidate:* Generate a candidate vector of parameters $\boldsymbol{\theta}^*_m$ such that

$$\theta^*_{l,m} = \begin{cases} \theta_{l,m-1} & \text{if } l \neq k, \\ \underline{\theta} \overset{iid}{\sim} \mathcal{N}\big(\theta_{l,m-1}, \sigma_{k,j}\big) & \text{otherwise,} \end{cases}$$

   for $l = 1, \ldots, n_k$.

3. *Accept-reject:* Set

$$\boldsymbol{\theta}_m = \begin{cases} \boldsymbol{\theta}^*_m & \text{accept with probability } \eta_m, \\ \boldsymbol{\theta}_{m-1} & \text{reject with probability } 1 - \eta_m, \end{cases}$$

   where

$$\eta_m := \min\left(1, \frac{p\left[\mathbf{Y} \mid \mathbf{f}(\boldsymbol{\theta}^*_m)^{-1}\right] p\left[\mathbf{f}(\boldsymbol{\theta}^*_m)^{-1}\right] J(\boldsymbol{\theta}^*_m)}{p\left[\mathbf{Y} \mid f(\boldsymbol{\theta}_{m-1})^{-1}\right] p\left[\mathbf{f}(\boldsymbol{\theta}_{m-1})^{-1}\right] J\big[\boldsymbol{\theta}_{m-1}\big]}\right),$$

   $\mathbf{f}$ and $J$ are defined below.

4. *Increase counter:* Increase $m$ by one.

- **Gibbs sampling**

For $j > 5000$ (burn-in period), use the univariate approach for multivariate time series of Koopman and Durbin (2000) to the simulation smoother proposed in Durbin and Koopman (2002) to sample the unobserved states, conditional on the parameters. In doing so, we follow the refinement proposed in Jarociński (2015).

- Burn-in period

  Discard the output of the first $j = 1, \ldots, 5000$ iterations.

- **Jacobian**

  As in Hasenzagl et al. (2022) most parameters are bounded in their support (e.g. the variance parameters must be larger than zero). In order to deal with this complexity, this manuscript transforms the bounded parameters ($\boldsymbol{\Theta}$) so that the support of the transformed parameters ($\boldsymbol{\theta}$) is unbounded. Indeed, the Adaptive Metropolis-Within-Gibbs draws the model parameters in the unbounded space. At a generic iteration $j$, the following transformations have been applied to a generic parameter $i$ with a Normal, Inverse-Gamma or Uniform prior:

$$
\begin{aligned}
\theta_{i,j}^{N} &= \Theta_{i,j}^{N} \\
\theta_{i,j}^{IG} &= \ln(\Theta_{i,j}^{IG} - a_i) \\
\theta_{i,j}^{U} &= \ln\left(\frac{\Theta_{i,j}^{U} - a_i}{b_i - \Theta_{i,j}^{U}}\right),
\end{aligned}
$$

  where $a_i$ and $b_i$ are the lower and the upper bounds for the $i$-th parameter. These transformations are functions $f(\Theta) = \theta$, with inverses $f(\theta)^{-1} = \Theta$ given by:

$$\Theta_{i,j}^{N} = \theta_{i,j}^{N}$$

$$\Theta_{i,j}^{IG} = \exp(\theta_{i,j}^{IG}) + a_i$$

$$\Theta_{i,j}^{U} = \frac{a_i + b_i \exp(\theta_{i,j}^{U})}{1 + \exp(\theta_{i,j}^{U})}.$$

These transformations must be taken into account when evaluating the natural logarithm of the prior densities by adding the Jacobians of the transformations of the variables:

$$\ln\left(\frac{d\Theta_{i,j}^{N}}{d\theta_{i,j}^{N}}\right) = 0$$

$$\ln\left(\frac{d\Theta_{i,j}^{IG}}{d\theta_{i,j}^{IG}}\right) = \theta_{i,j}^{IG}$$

$$\ln\left(\frac{d\Theta_{i,j}^{U}}{d\theta_{i,j}^{U}}\right) = \ln(b_i - a_i) + \theta_{i,j}^{U} - 2\ln(1 + \exp(\theta_{i,j}^{U})).$$

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