Heterogeneous Market Beliefs, Fundamentals and the Sovereign Debt Crisis in the Euro Zone

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Abstract

The unprecedented sovereign debt crisis across the Euro Zone has prompted a new generation of models with "self-fulfilling" attacks to public debt. The model presented in this paper has three main features: (i) the government's default decision arises out of a cost-benefit analysis that sets the sustainable limit of the solvency primary balance; (ii) investors have no direct information about this variable but form individual rational beliefs, and (iii) the debt market is characterized by the frequency distribution of their beliefs. Multiple equilibria are possible and the model identifies an attraction domain of default within which the government is bound to default although initial solvency conditions are satisfied. Then several issues are discussed concerning the role of initial conditions, fiscal shocks, and the policy options to escape from the default domain.

Keywords: Models of public debt, speculative attacks, euro-sovereign debt crisis

JEL Codes: E6, H6

1. Introduction

(...) we are in a situation now where you have large parts of the euro area in what we call a "bad equilibrium", namely an equilibrium in which you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios. So, there is a case for intervening, in a sense, to "break" these expectations (...) But then, we should not forget why countries have found themselves in a bad equilibrium to start with (Draghi (2012, p. 4)).

This quotation from the presentation of the European Central Bank's (ECB) "Outright Monetary Transactions" new programme for purchases of government bonds certifies the official endorsement of a new "multiple equilibria" (ME) approach to sovereign debt analysis. This approach marks a substantial modification of theory and policy with respect to the orthodox view of "market discipline" and "credibility", based on the efficient market hypothesis with single rational-expectations equilibrium, traditionally endorsed by the European institutions.

The thrust of models with ME is that (at least) one possible equilibrium is the result of "self-fulfilling prophecies", that is agents' beliefs about future states of the economy that turn out to be true in force of the beliefs themselves. Self-fulfilling (SF) prophecies are a long-standing research field (e.g. Farmer (1993)). Financial and currency markets are natural fields where this class of models has proved able to provide valuable insights into complex phenomena such as bubbles, crashes, or speculative attacks. In fact, the closest antecedents to the ME approach to sovereign debt date back to the various "generations" of models of currency crisis and exchange-rate regime collapse of the 1980s and 1990s (e.g. Obstfeld (1995)). Early extensions to sovereign debt also appeared with special reference to emerging economies (e.g. Calvo (1988), Cole and Kehoe (2000)), but they are now being boosted by the dramatic euro-sovereign debt crisis that erupted in Greece in early 2009 and then propagated across the whole area.

ME models may have an intrinsic theoretical interest and motivation, and this paper is no exception. However, this new wave of studies seeks to address and explain in a consistent framework a set of phenomena in the Euro Zone (EZ) sovereign debt crisis that has rapidly grown to challenge the orthodox view:

- there is scant evidence of consistent "market discipline", that is, the correct "fundamental" pricing of bonds, *throughout* the life of the euro:

typically, (some) country risk spreads were too low until 2008; they have been too high since 2009 (Di Cesare et al. (2012))

- there is evidence that post-2009 spreads not only reflect country-specific fundamentals, but are also highly sensitive to "systemic risk" and other exogenous factors (Manganelli and Wolswijk (2009), Sgherri and Zoli (2009), Attinasi et al. (2009), Caceres et al. (2010), Favero and Missale (2011))

- there is evidence of "contagion", that is, the transmission of high spreads across countries via non-fundamental channels (Caceres et al. (2010), De Grauwe and Ji (2012), Tola and Wäldi (2012))

- there is evidence of SF processes via the positive feedback mechanism among market beliefs of default, higher spread, higher fiscal effort, reinforcement of market beliefs (De Grauwe and Ji (2013))

With regard to these phenomena, a key feature of ME models of sovereign debt crisis is that fundamental fiscal variables and market beliefs interact, so that one possible equilibrium is typically a SF default due to the positive feedback mechanism described above. Hence, a sovereign may be driven to default even though it is solvent in initial conditions. Along this perverse trajectory, contrary to fiscal orthodoxy, attempts at strong fiscal consolidation may be counterproductive. The possibility of this scenario has of course important policy implications.

Like ME models of currency crisis, also those of sovereign debt crisis now display different "generations". An earlier generation of models (e.g. Adrian and Gros (1999)) was concerned with the optimal choice of instruments whereby the government *can always remain solvent*, typically taxation or monetization (inflation). Following the seminal paper by Calvo (1988), the current generation of models is concerned with institutional set-ups where the government is constrained in the use of these instruments (for instance, EZ governments have no access to monetization) and therefore *it can in fact opt for default* (e.g. Cooper (2012), Corsetti and Dedola (2011), Gros (2012), De Grauwe (2011), Ghosh et al. (2013)).[1] This approach is supported by extensive historical evidence showing that default is almost always a government choice not necessarily forced by immediate inability to pay (Tomz and Wright (2013), Buiter and Rahbari (2013)).

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[1] De Grauwe (2011) and De Grauwe and Ji (2012) stress the different behaviour of risk premia *vis-à-vis* fiscal fundamentals for countries within and outside the EMU.

The earlier model by Calvo was deterministic, one where all agents are perfectly informed about the government's choice model, there is no uncertainty and hence the no-default and default equilibria are rational-expectations (RE) equilibria. Corsetti and Dedola (2011) and Cooper (2012) introduce uncertainty into the Calvo model in the form of an exogenous random shock to the government's ability to pay such that it opts for default[2]. This kind of "exogenous uncertainty" is also adopted by Ghosh et al. (2013). Therefore, this class of ME crucially depends on the characteristics of the probability distribution of shocks, while the RE hypothesis still holds in that investors know the true probability distribution.[3] An interesting variation is proposed by Gros (2012). He introduces uncertainty by way of the political process leading to the government's default decision, such that a higher cost of solvency increases the probability that the pro-default party wins, though this result is not certain owing to other factors. Again, this probability information is known to investors who rationally use it in the calculation of the default premium, so that all ME are RE equilibria.[4] This approach to uncertainty seems both more interesting and consistent with the focus on the government's decision process than assuming exogenous shocks to the government's ability to pay.

Here I present a ME model of sovereign debt crisis belonging to this latter generation, which fits the EZ institutional features quite easily. The model focuses on the interplay between the government and investors and hinges on three key characterizations. First, in each period of time the government, given the relevant economic conditions among which the interest rate on outstanding debt, faces a solvency condition to be accomplished in the next

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[2] An adverse shock may hit output, so that it may be too low to generate the no-default tax revenue.

[3] In the model by De Grauwe (2011), it is the expectation of default itself that raises the risk premium and makes solvency too costly in the event of a shock. Hence, default is a RE equilibrium but in the sense that expectations are self-fulfilling though unrelated to fundamentals or objective probability distributions. However, the model does not explain how default or non-default expectations are formed, or how investors coordinate on one expectation instead of the other. Also, the result is problematic since in reality governments may find it profitable to default when this is unexpected, and some investors do remain entrapped, rather than the other way round.

[4] One key finding stressed by Gros is that the region of multiple equilibria is smaller than in the case of certainty, which seems a counterintuitive result. As will be seen, this is not necessarily the case in the present model.

period. The solvency condition is simply the primary-budget/GDP ratio or "fiscal effort" $b^* > 0$ necessary to keep the debt/GDP ratio constant (e.g. Ghosh et al. (2013), Buiter and Rahbari (2013)). Its decision of solvency or default is the result of a *cost-comparative analysis setting a threshold level of fiscal effort* $\bar{b} > 0$ beyond which default is preferred. The government commits itself with the market to achieve b^*, and it will comply as long as $b^* < \bar{b}$; but, as will be seen, there is no credible communication of $\bar{b}$. Hence the problem is on the investors' side. Second, investors are risk neutral and operate under *uncertainty about the default event* because they understand the government's choice-theoretic setup, but they have no access to the full information necessary to know the true default threshold $\bar{b}$. However, I depart from the fiction of the representative investor endowed with "the" probability distribution of default. As the third and novel feature of the present model, investors elaborate *heterogeneous (rational) beliefs about* $\bar{b}$. Hence, for any given b^* to which the government is committed, each investor decides whether to invest in the debt market or in an alternative safe asset depending on b^* being lower (sustainable) or higher (non sustainable) than his/her belief about $\bar{b}$. The market operates as an "aggregator of beliefs", and the determination of the interest rate depends on the *characteristics of the distribution of beliefs.*

Heterogeneous beliefs (HB) provide an alternative approach with respect to the standard paradigm of efficient financial markets based on the RE and representative agent hypotheses. This choice has various motivations. It is hardly disputable that "heterogeneous beliefs are a fact of life" (Xiong (2013, p. 14)), and abundant evidence comes from the economic profession itself. HB existence, persistence and economic relevance are by now largely documented and measured rigorously by empirical investigations of professional forecasts in various fields (e.g. Mankiw et al. (2004), Wieland and Wolters (2011), for macroeconomic variables) which now also cover fiscal forecasts (Poplawsky-Ribeiro and Rülke (2011)). A three-decade theoretical debate has shown that, to say the least, the conditions underpinning the representative agent *cum* RE cannot be taken for granted, and they may be more the exception (to be proved to exist) than the rule (e.g. Kurz (2011), Frydman and Phelps (eds., 2011), Xiong (2013)). In Kurz's (2011) words, "lack of knowledge of the truth is the foundation of belief diversity" (p. 191). According to a long-standing line of thought in the sciences of human knowledge, inability to attain "the truth" about

experience phenomena, and to attain it *unanimously*, is rooted in our cognitive limitations *vis-à-vis* the complexity of the external environment, so that "all the circumstances which will determine the outcome of a process […] will hardly ever be fully known or measurable [...]" (Hayek, 1974, p.3). Hence, moving from an individual, partial and conjectural basis, beliefs may well differ across agents though being formed according to rational procedures.[5]

Fiscal policy matters seem particularly prone to heterogeneity of beliefs. The multidimensional complexity of fiscal sustainability assessment (e.g. Kanda (2011), IMF (2012), Ghosh et al. (2013)), and the political complexity of the governments' choice process evoked by Gros (2012), combine in a good case in support of Kurz's claim. Furthermore, default is a rare or unique event in each specific country for which "objective" inferences based on recurrent observations are not available (before Greece, no default had occurred in Western Europe in sixty years). Indeed, Poplawsky-Ribeiro and Rülke (2011) find persistence of biases and heterogeneity in professional fiscal forecasts[6], though these have been reduced for countries subject to the SGP rules, which have probably restricted the degree of variability of fiscal policies across countries. Alsakka and ap Gwilym (2010) report that inputs, techniques, practices and outcomes differ across credit rating agencies more widely in the sovereign sector than in the corporate sector. Nonetheless, the implications of HB regarding the emergence of ME with SF default in the sovereign debt market are relatively unexplored.

There are various characterizations and modelling techniques of HB in the literature. In 2.4 I expound my specific assumptions, which aim to set the stage at the most tractable level as a first step of analysis. Investors form individual, independent and private beliefs about $\bar{b}$, which are rational (in the sense that will explained) and invariant in the market

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[5] For a cognitive approach see Tamborini (1997). Another approach, particularly suited to professional agents and the market for forecasters and advisors, points out that acquisition of knowledge and information is a costly activity from which an individual competitive advantage is expected only if the results remain exclusive private goods (see the classical Grossman and Stiglitz (1980)). Hence diversification of knowledge and information is not just a "friction" imposed on agents from outside, but it is the result of conscious activity responding to economic incentives

[6] Consensus Economics Forecasts (www.consensuseconomics.com) elaborates on a monthly basis professional forecasts of fiscal deficits for major European countries for the current year and one year ahead.

process (i.e. they are treated like preferences). These beliefs about $\bar{b}$ can be represented in a frequency distribution (think of those commonly used by surveys of professional forecasts), unobservable to each single investor, which is the key element in the model.

The model embeds the positive feedback mechanism that characterizes SF beliefs between the government's solvency condition and the market interest rate arising from investors' beliefs about its default threshold. In fact, by way of the distribution of beliefs, for any given b^* to which the government is committed, there is a fraction of "pessimist" investors who believe that b^* exceeds the default threshold, and the complement fraction of "optimists" who believe it does not. I then obtain that the interest rate is increasing in b^*, since the larger is b^*, the larger is the fraction of pessimists who wish to sell, and the smaller is the fraction of optimists who wish to buy. On the other hand, for the government the higher is the interest rate, the larger should be b^*. An equilibrium is a mutually consistent couple (b^*, i^*). There can be two non-default states, a "good equilibrium" (stable) with low fiscal effort and interest rate, and a "bad equilibrium" (unstable) with high fiscal effort and interest rate, and a third default states. Though able and willing to stay solvent at initial conditions, the government may eventually be induced into default because too large a fraction of investors so believe.

As will be seen, the HB hypothesis allows for a richer analysis with new insights. In the first place, underlying interest-rate changes are *actual trades in the market*, and what matters is the relative fraction of pessimist and optimist investors. Consequently, all equilibria are *a joint product* of fiscal fundamentals and market beliefs: the two components cannot be disentangled meaningfully. In particular, the nature of the equilibria, and the extension of the domain of attraction of default crucially depend on the *first two moments* of the distribution of beliefs, a typical feature of HB models (Kurz (2011)). It will be seen that this implies that the distribution of beliefs cannot consistently be inferred from market realizations, which supports the assumptions on the formation of beliefs. Further, not only fiscal shocks, but also market shocks matter, that is changes in the distribution of beliefs whether due to fundamentals or not. The model also helps shed light on some puzzling phenomena of debt crises, in particular those observed in the EZ, and their policy implications. Overall, phenomena in contrast with the efficient market hypothesis are not necessarily due to

brute irrationality, but to heterogeneous agents who operate in a more complex environment of which none "knows the truth", and who act consistently with their individual beliefs based on the limited knowledge and information they have.

The paper is organized in two parts. The model is expounded and discussed with reference to the relevant literature in section 2. Section 3 of the paper shows the model at work, covering the distinction between fiscal and market shocks, large and small, puzzles in spread patterns across countries and over time, domestic *vs.* foreign debt, why "austerity" may not work. The aim of this section is not to provide detailed policy solutions or examine those under discussion in the EZ, but only to show how the model can be used to frame policy analysis to be further developed. Summary and conclusions follow in section 4.

2. The model

2.1. Basic notions

To begin with, let us examine the evolution of public debt[7] over time in a forward-looking perspective from the current year t. For concreteness, think of the budget plans that the EZ governments should submit to the Commission. Let all relevant variables be known in advance. The nominal value of debt in $t+1$ D_{t+1} will be

(1) $D_{t+1} = D_t + (I_{t+1} - B_{t+1}) - M_{t+1} + X_{t+1}$

that is, the value of outstanding debt at the end of t D_t, plus the government's net borrowing (in brackets), minus central bank's direct lending ("monetization" for short, M_{t+1}), plus extraordinary debt operations and other corrections X_{t+1} (often called "stock-flow adjustments", see e.g. EU Commission (2011)). In the present context, it will be convenient to assume that the government can freely decide an extraordinary operation of debt creation $X_{t+1} > 0$ (e.g. a swap of public for private debt), but it cannot freely

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[7] Debt held by the resident and non-resident private sector. Excluded is the debt held by other public institutions, namely the central bank. In fact, interests paid on the latter share of debt do not constitute net disbursements for the public sector as a whole.

do the opposite unless it defaults by the amount $X_{t+1} < 0$.[8] The government's net borrowing in $t+1$ will result from the difference between the service I_{t+1} of the outstanding debt and the primary balance B_{t+1}. The usual simplifying shortcut is that debt is serviced with delay in consideration of maturities etc., so that $I_{t+1} = \theta_t D_t$, where θ_t is the average cost of debt. Therefore,

$$(2) \qquad D_{t+1} = (1 + \theta_t)D_t - B_{t+1} - M_{t+1} + X_{t+1}$$

In each t a constant share e of debt expires, of which a share r can be rolled over; hence D_t is the result of the outstanding debt of the previous year minus expirations plus rollovers. If rollovers match expirations ($r = 1$), the average cost of debt is $\theta_t = (1 - e)\,\theta + ei_t$. Note that the average cost of debt changes over time only if the marginal cost (the interest rate i_t) paid on rollovers differs from the historical average cost θ. If i_t remains constant, in the long run (depending on the rate e), $\theta = i_t = \theta_t$. Thus the short-run θ_t contains a crucial information about the future cost of debt.

To keep the treatment manageable, I consider the (minimal) solvency requirement of constant debt/GDP ratio (e.g. Ghosh et al. (2013), Buiter and Rahbari (2013)). Denoting ratios of nominal fiscal variables to GDP with small-case letters, the dynamic equation of the debt/GDP ratio is

$$d_{t+1} = \frac{1 + \theta_t}{1 + z_{t+1}} d_t - b_{t+1} - m_{t+1} + x_{t+1}$$

where z_{t+1} denotes the rate of year change in the nominal GDP. Recall that $b_{t+1} = \tau_{t+1} - g_{t+1}$, where τ_{t+1} is the average tax rate and g_{t+1} is the public expenditure in goods and services relative to GDP. Hence, in order to keep the debt/GDP ratio constant, $d_{t+1} = d_t = \bar{d}$, the following solvency constraint should hold at all times:

$$(3) \qquad \frac{(\theta_t - z_{t+1})}{(1 + z_{t+1})}\bar{d} - (\tau_{t+1} - g_{t+1}) - m_{t+1} + x_{t+1} = 0, \;\; x_{t+1} \geq 0$$

We are now in a position to appreciate the special status of a sovereign. Given $\{\bar{d},\; \theta_t,\; z_{t+1}\}$, a sovereign *can always choose* the appropriate combination in the set of control variables $\{\tau_{t+1},\; g_{t+1},\; m_{t+1},\; x_{t+1}\}$ that satisfies the solvency constraint at any time. Two are the variables that make the difference with any other ordinary debtor. One is the tax rate τ_{t+1},

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[8] Alternatively $X_{t+1} < 0$ may denote debt purchase by the central bank in the secondary market, since this moves a fraction of the outstanding debt out of the private sector (see fn. 7).

and the other is the monetization rate m_{t+1}. In fact, by imposing taxation, a sovereign can raise its revenues, while by monetization it can expand its ability to pay, in ways that are precluded to ordinary debtors. The cost-benefit comparison of these two instruments was the focus in the first generation of debt models. Where does a sovereign's solvency problem come from? It may come from constraints imposed, or self-imposed, on its ability to manipulate the above set of variables at will. This is the key to the second generation of models of the government's default choice.

2.2. The default decision

The case of EZ sovereigns is emblematic, since they face the institutional constraint $m = 0$ at all times. In order to stay solvent these governments are compelled to achieve a level of the primary balance/GDP ratio b^*_{t+1}, that is a combination of $\{\tau_{t+1}, g_{t+1}\}$, that fulfils the solvency constraint, all other variables being given. Since for normal order of magnitude of z, $1 + z \approx 1$, the solution of b^*_{t+1} for the solvency constraint (3) is usually approximated by

(4) $b^*_{t+1} \approx (\theta_t - z_{t+1})\overline{d} + x_{t+1}$

If the government fulfils the solvency constraint at all times, the debt/GDP ratio remains constant and the debt stock should be willingly held by investors at the equilibrium market interest rate i^*. This entails that rollovers are regularly refinanced at the rate i^*, so that in the long run $\theta_t = \theta = i^*$. Anticipating this, solvency is best seen by the government as a long-run steady-state equilibrium (time subscripts are dropped) such that at all times $\{m, x, z\} = 0$, and

(5) $b^* = i^*\overline{d}$.

is the long-run solvency primary surplus.

Now the point is that, along the solvency path, the larger is the solvency primary surplus, the more the government should either raise taxes or cut expenditure. Both options involve welfare costs and/or political costs. These may also include effects on GDP that feed back onto the solvency constraint. It should also be pointed out that, as will be seen, the existence of ME and the main features of this model *do not depend on these effects.*

In this perspective the default decision is not necessarily dictated by "objective" financial factors, but it essentially depends on the comparative costs of the various options that the government faces. Many models are available, usually based on the optimization of some objective function of the

government (see Introduction). Also, default may in practice take a variety of forms and extensions. A critical factor is the amount of cut on the principal due to creditors, the so-called "haircut", which in itself should be a choice variable in the cost-comparative problem of the government (e.g. Gros (2012)). These technicalities would complexify the analysis in a substantial way, but they are not essential, and I shall keep them out of the model. I assume as known that default consists of total debt repudiation (e.g. Calvo (1988)).

For the present purposes, it is sufficient to assume that the cost of solvency is increasing and convex in b^*, $\Phi(b^*) > 0$, $\Phi'(b^*) > 0$, $\Phi''(b^*) \geq 0$. On the other hand, the government also perceives costs from default. These are generally related to serious damages to the economy (e.g. negative wealth effects to domestic bond-holders, disruption of financial and credit institutions) as well as the political loss of reputation towards electors and creditors that may thwart future re-election and access to borrowing. These costs are likely to be perceived as independent of the size of the budget (debt), $\Theta(b^*) > 0$, $\Theta'(b^*) = 0$, as well as of the size of default or of other technicalities. This comparative-cost framework is sufficient to obtain a default rule.

In fact, given b^*, the government will always choose $\min(\Phi(b^*), \Theta(b^*))$. Yet there exists a single value $\bar{b}$ such that (i) $\Phi(\bar{b}) = \Theta(\bar{b})$, and (ii) $\Phi(b^*) \gtrless \Theta(b^*)$ for any $b^* \gtrless \bar{b}$. Hence the government will comply with b^* only up to the threshold primary-surplus $\bar{b}$ beyond which the cost of solvency exceeds the cost of default. Note that $\bar{b}$ is increasing in the cost of default and decreasing in the cost of solvency.

It should also be borne in mind that in reality the government's options include not only solvency/default but also partial fiscal adjustment, i.e. a primary surplus $b < b^*$. In this (frequent) case, the consequence is usually not immediate default, but rather an increase in outstanding debt at a higher interest rate that defers either full solvency with greater fiscal effort or default. This entails an intertemporal cost assessment that I will not consider here.

Against this background, I now introduce the government's reaction function (GR), $b^*(i)$, which yields the solvency primary surplus b^* in response to any interest rate i set by the market, to which the government

commits itself, and which it will achieve up to $\overline{b}$. In view of long-run solvency, the GR function is[9]

(6) $b* = \begin{cases} i\overline{d} \leq \overline{b} \\ \text{default otherwise} \end{cases}$

2.3. Probability of default and sovereign default premium

I now move to the investors' side, which I treat symmetrically with the government, that is an investors' reaction function (IR), $i(b*)$, that yields the market interest rate in response to any value of $b*$ communicated by government. Investors may choose between holding the government bond at the market rate i or an alternative safe asset yielding a constant return $\overline{i}$. They know that, at maturity, the bond yields $(1 + i)$ per unit of capital if the government is solvent or zero if it defaults. They are risk neutral but operate under uncertainty about the value of the default threshold $\overline{b}$, and hence whether the government will comply with $b*$ or not. On the other hand, as will be explained later (see 3.1), there is no credible communication of the true value of $\overline{b}$ by the government *ex ante*.

As a benchmark, I first introduce the standard model of determination of the interest rate on the risky bond, that is one with the risk-neutral representative investor who holds "the" probability distribution of possible default thresholds $f(\tilde{b})$ and discounts the probability p of default thresholds below $b*$. Given p, the uncovered interest parity condition yields

(7) $1 + i = \dfrac{1 + \overline{i}}{1 - p}$

Note that, as commonly expected, i is increasing in p; $i - \overline{i}$ is the ensuing default premium, which is zero for $p = 0$ and increases asymptotically as $p \to 1$.

Here I propose a different representation of the market structure and of interest rate determination departing from the representative agent and allowing for heterogeneous investors in their beliefs about $\overline{b}$.

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[9] This reaction function can also be interpreted as a stepwise "all-or-nothing" case of the S-shaped function of "fiscal fatigue" employed by Ghosh et al. (2013), which instead postulates a smooth transition from compliance to non-compliance with the solvency condition.

2.4. Introducing heterogeneous beliefs

In the light of the existing literature, I design HB in the simplest and most tractable way, as a first step of analysis. First, investors form individual rational beliefs about $\bar{b}$. Beliefs are rational, in the broad sense that they are formed consistently with the government's choice-theoretic model generating the default event, except the exact dimension of $\bar{b}$. For simplicity (but not necessarily) each individual belief is treated as a point value. Second, beliefs are independent and private knowledge. Third, beliefs can be represented in a frequency distribution (think of those commonly used by surveys of professional forecasts), but since no individual investor has access to the beliefs of others, their distribution is not known either (it will also be seen that the distribution cannot consistently be inferred from market outcomes). Fourth, beliefs and their distribution remain invariant in the market process, though I will discuss the implications of exogenous changes in the distribution.

As ordinary people do not think (correctly) that diversity of beliefs about economic events is symptom of irrationality, so economic models with HB should be anchored to some standard of rationality. There are various characterizations of rational beliefs in the literature. The minimal requirement is generally that beliefs are somehow connected to what agents can know and observe about the relevant economic event(s) (Kurz (2011)). My chosen characterization belongs to the broad category of "model consistent beliefs" (where "model" stands for educated knowledge of the relevant economic process). Model consistency is the same foundational principle of the RE hypothesis, but the postulate that all agents come to know the unique "true" model is dropped. The intuition is that investors correctly understand the government's default choice model, but no one possesses full knowledge and information of all the specific inputs leading to the actual level of $\bar{b}$, which are open to subjective, diversified assessment. Let Ω be the complete set of determinants of $\bar{b}$. Let it be decomposable in subsets $\omega_n \in \Omega$, with mappings F_n from ω_n to $\bar{b}_n$. Subsets ω_n may differ either because each contains (some) different elements from the others and/or because each contains different measures of the same elements. Hence, individual beliefs (and decisions) are all consistent with the process generating the default event except the exact dimension of $\bar{b}$. As will be seen in 3.1, an important implication is that as (some) determinants of $\bar{b}$ in

Ω change (or are supposed to change), also the *distribution* of beliefs changes accordingly.

The other three assumptions essentially define beliefs as individual primitive characteristics like preferences, and exclude more complex structures such as "second order beliefs" (i.e. beliefs on the beliefs of others) and endogenous modifications of beliefs (e.g. learning, induction, eduction). As to the first limitation, normatively compelling as full information may be, it is not so obvious that individual agents' information sets *can* contain, or *should* contain, information on the beliefs of all the others. Whether each individual can attain knowledge of the beliefs of all the others or not, according to a long-standing methodological view consistent analyses of market structures with a large number of "atomistic" individuals who cannot alter the market conditions by their own actions should in the first instance rule out that individuals can communicate, or come to know, each others' information, beliefs, etc., *ex ante*. The coordination of choices of heterogeneous individuals is the critical task of the market itself (Hayek (1945), Frydman (1983)). As to the second limitation, it may be seen as injecting persistence of HB by brute force. An important strand of literature has explored systems where individual beliefs change endogenously in the market process, as they are updated in the light of the system's outcomes, which may also convey signals on the beliefs of others. It has been shown that heterogeneity may persist, and that conditions underpinning convergence in beliefs cannot be taken for granted (for recent reviews see Frydman and Phelps (eds., 2011), Xiong (2013, pp. 16-ff.)). Since persistence of HB is also largely documented by empirical investigations, my assumption may be convenient as a first approximation.

Now imagine that we (as meta-observers) have the investors' opinion poll about the level of $\bar{b}$ so that we can construct the relative frequency distribution of such beliefs $\bar{b}_n$ as in common surveys of professional forecasts. Given b^*, each investor will hold the government bond or not according to whether his/her $\bar{b}_n$ is greater or lower than b^*. For mathematical convenience, let us consider a continuum of individual beliefs denoted $\tilde{b} \in [\tilde{b}_{\text{inf}}, \tilde{b}_{\text{sup}}]$, $\tilde{b}_{\text{inf}} > 0$, with distribution $f(\tilde{b})$. The average, or "market belief" about $\bar{b}$ is

(8) $$\bar{b}_M = \int_{\tilde{b}_{\text{inf}}}^{\tilde{b}_{\text{sup}}} \tilde{b} f(\tilde{b}) \mathrm{d}\tilde{b}$$

As a (strong) implication of the hypothesis that beliefs are rational, we may (though not necessarily) posit the *cross-sectional* restriction $\bar{b}_M = \bar{b}$ – i.e. the market is right as belief aggregator.

We can now obtain the IR function, $i(b^*)$. For any given b^*, we can compute the cumulated fraction of investors who believe that the government will default – call them "pessimists" – that is the subset of beliefs $\tilde{b} \leq b^*$ with measure

(9) $\qquad F(b^*) = \int\limits_{\tilde{b}_{inf}}^{b^*} f(\tilde{b}) \mathrm{d}\tilde{b}$

$F(b^*)$ is therefore the fraction of investors who wish to switch from the risky bond to the safe asset. The bond price should fall and the interest rate rise. On the other hand, the complement fraction of investors $1-F(b^*)$ with $\tilde{b} > b^*$ – call them "optimists" – wish to remain in, and are willing to buy, the risky bond for any non-zero default premium.[10] The ensuing level of the interest rate depends on the relative dimension of the two fractions at b^*. A feature of the cumulated distribution is that it increases monotonically with b^*, $F'(b^*) > 0$. Hence, as b^* increases, the fraction of pessimists grows and that of optimists shrinks. We thus have a mechanism of interest rate determination based on sales from pessimists to optimists such that the interest rate is increasing in b^*, $i'(b^*) > 0$. Note two important qualifications. As $F(b^*) \to 0$, no investor believes that the government will default and hence all investors are indifferent between the risky bond and safe asset so that $i = \bar{i}$. As $F(b^*) \to 1$, all investors believe that the government will default, there is no longer market for the risky bond and interest rate should rise asymptotically.

Though not strictly necessary, it is convenient to look for a specific functional form of the IR function consistent with the general features discussed above. A good candidate is still equation (7) with $p = F(b^*)$:

(10) $\qquad i = \dfrac{1 + \bar{i}}{1 - F(b^*)} - 1$

In this context, this formulation has no longer the normative content of the single agent model, because no individual in the market knows the true probability distribution of beliefs, and each holds his/her belief of default or

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[10] The marginal investor who splits the population at point b^* remains indifferent between the risky bond and the safe asset. By way of the increase in the interest rate, the bond holders receive a premium very much like the consumer surplus.

no-default with probability 1. Nonetheless, it has simple and suitable descriptive properties.

The fraction $F(b^*)$ of investors who believe that the government will default is indeed the probability of default expressed by the market as a whole – I will call it the "market" probability of default. As in the Hayekian tradition, the market operates, *inter alia,* as an "aggregator of beliefs", given that nobody has full knowledge of all individual beliefs and of their true distribution. With regard to the transmission of this information via the interest rate, we shall see in 3.1 that what does matter are the first two moments of the distribution of beliefs. As a consequence there may be two different distributions that determine the same i for the same b^* (see Figure 6), which has the important implication that no individual investor can consistently infer the distribution of beliefs from observed couples (b^*, i). This supports the initial assumptions on the formation of beliefs. Note, also, that the function (10) fulfils the qualifications pointed out above for $F(b^*) \to$ 0 and $F(b^*) \to 1$, while in-between it grants that i will be monotonically increasing in b^*. I therefore adopt (10) as the IR schedule of interest rates at which the existing stock of debt is willingly held, and rollovers regularly match expirations, for any b^* communicated by the government.

It might be argued that, apparently, the same result is obtainable through the shortcut of the representative agent endowed with the true distribution function $f(\tilde{b})$. Apart from the well-known foundational problems recalled previously, heterogeneity has at least one notable advantage: the interest rate is the result of *trading* triggered by different beliefs about the sustainability of b^*. This is consistent with, and may provide useful insights into, the ordinary working of markets of risky assets (e.g. why do supposedly rational investors remain entrapped into the default?) that can hardly be accommodated with the representative agent with RE. The applications presented below and in section II will give the opportunity to return to these issues.

2.5. "Good" and "bad" equilibria

The thrust of the previous treatment consists of three elements: (i) the government's threshold value $\bar{b}$ of the default decision, (ii) the GR function (6), (iii) the IR function (10). The key feature of the GR-IR system is that b^* and i are interdependent via the function $F(b^*)$. Given the properties of the latter, the result is the typical positive feedback mechanism such that

the higher is b^* the higher is i, and so forth.   In order to study this system, in particular the existence of ME, we need to examine the GR and IR functions in greater detail.

The GR function is linear and increasing in i; IR is monotonically increasing and nonlinear in b^* for any distribution of beliefs. Hence ME may exist. The curvature of IR is also important in order to establish the properties of the system, yet the sign of the second derivative of the function cannot be established in general without knowing the underlying distribution.[11] Nonetheless, a sensible restriction is that IR is strictly convex. Apart from mathematical considerations[12], the convexity of the IR function is suggested by the observed relationship between (the logs of) spreads and b^* in the EZ countries during the climax of the sovereign debt crises 2010-12 (see Figure 1 and Figure 2). The economic meaning of convexity offered by our IR function is that the translation of greater fiscal efforts into higher interest rates takes place "smoothly", though at an accelerating pace, by way of sales of growing pessimists to shrinking optimists. Even when rumours of default were very high, demand for Greek, Spanish or Italian bonds became thinner but never vanished. And, actually, some investors borne the losses of the Greek default.

Figure 1. Average monthly spreads and b^* values for EMU11 countries, 2010-12
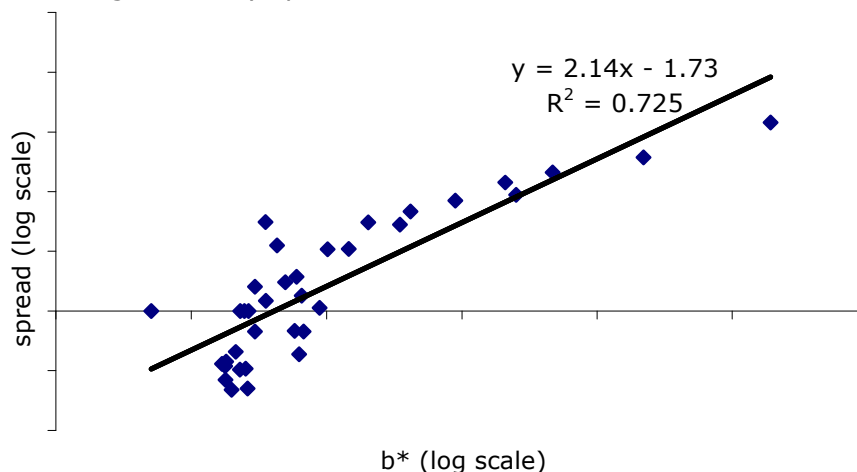
Average monthly spread: year average of the monthly difference between the yield rate of long-term bonds and the German *Bund* (ECB, Interest rate statistics)
b^* values: author's calculations based on equation (3) (Eurostat, AMECO database)
EMU11: early accession countries up to Greece, except Germany.

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[11] $\text{Sign}(\partial^2 i/\partial b^{*2}) = \text{sign}(F''(b^*)(1-F(b^*)) + \bar{i}\, F'(b^*))$.

[12] Concavity of IR would have the implausible implication that as b^* grows i increases with decreasing intensity.

Figure 2. Average monthly spreads and $b*$ values for most distressed Euro-countries, 2010-12
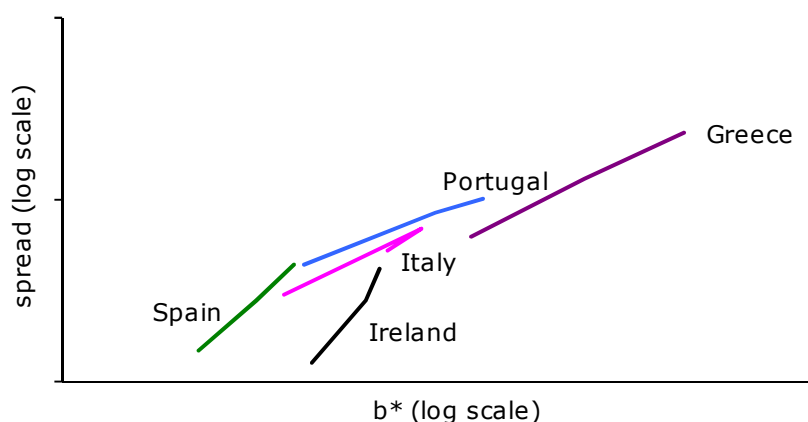
Source: see Figure 1.

There are at least two known distributions which generate a strictly convex IR: the Uniform and the Normal.[13] Figure 3 exemplifies the IR function generated by a Normal distribution of beliefs, and $\bar{i} = 2\%$.[14]

Figure 3. The IR function with a continuous Normal distribution N~(7, 1.4) of the investors' beliefs, $\bar{i}$ = 2%
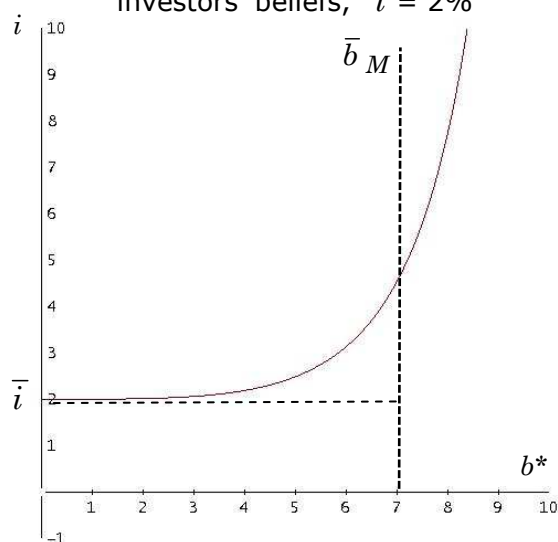
Normality is a (sensible) case where opinions are relatively concentrated around the market belief $\bar{b}_M$ with tails of optimists and pessimists. Note

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[13] For the Uniform distribution, $F''(b*) = 0$, so that $\partial^2 i / \partial b*^2 > 0$.

[14] The other parameters are $\bar{b}_M = 7$, $\sigma = 1.4$, $\tilde{b}_{inf} = 2$, $\tilde{b}_{sup} = 10$. The probability mass contained within the domain of beliefs is 98.38%

that the function is almost flat (the default premium is negligible) for a relative wide range of low values of b^*; i increases faster as b^* approaches and then exceeds $\bar{b}_M$. This property indicates that the pace of increase of the interest rate does not depend on the level of b^* *per se* but on its *distance from the market belief* $\bar{b}_M$. [15]

We can now examine the properties of the GR-IR system in the space (b^*, i) in Figure 4. On the GR function we can read the value of b^* to which the government commits itself for any given i. On the IR function we can read the value of i set by the market for any b^* communicated by the government. An equilibrium is a couple (b^*, i^*) such that (6) and (10) are verified simultaneously (i..e. it is a fixed point solution).

Figure 4. The GR-IR model
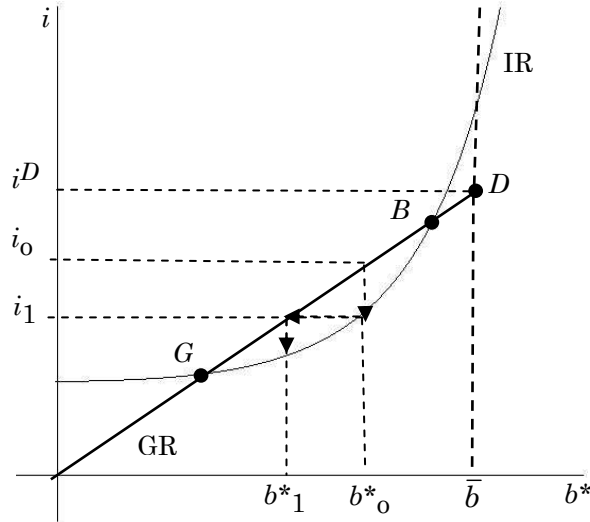
The geometry of these functions allows for ME. As long as $\bar{i} > 0$ and $F(\bar{b}) < 1$, which I regard as the normal cases, equilibria can be up to two. Figure 4 represents this case: G is a "good" equilibrium (low b^* and i^*), B is a "bad" equilibrium (high b^* and i^*). D is the default state. Note that, unlike ME models with RE, the bad equilibrium is not necessarily the default state, and the default state is not necessarily a fixed point; this may be the case

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[15] To see why the heterogeneity hypothesis, that is a nondegenerate belief distribution, is crucial in this connection, consider what would happen if beliefs collapsed on their mean value. The IR function would become the rectangular function with dotted lines in Figure 3. That is to say, the spread would remain zero for all b^* up to $\bar{b}$ where it would jump to infinity.

only if IR intersects GR exactly at i^D, the interest rate that triggers default. Nonetheless, default remains a possible event driven by investors' beliefs, given that the government cannot sustain a cost of debt greater than i^D. Note that even when the government defaults some investors do remain entrapped and bear losses. In fact, if the true default threshold is contained in the beliefs' domain, $\bar{b} < \tilde{b}_{\text{sup}}$, the fraction of investors long in the bond when the government defaults is $1 - F(\bar{b}) > 0$. Concomitantly, default occurs at a high, but finite, level of the interest rate.

As standard practice in ME models, though limitative, I now examine the local properties of equilibria independently of a specific characterization of the system's behaviour out of equilibrium (e.g. its dynamic structure: the order of actions of government and investors, and of the ensuing market realizations). This would require additional assumptions that are left for further developments, whereas in what follows I will give a logical account of how the positive feedback mechanism works.

Let us examine points G and B. If G exists, it should be that $\partial \text{IR}/\partial b^*|_G < \partial \text{GR}/\partial b^*|_G$, which means that G is an attractor. Let us consider an initial arbitrary value of i_0 to the right of G as in Figure 4. The corresponding solvency primary surplus is b^*_0. But the IR function indicates that for b^*_0 the market would demand a lower i_1, which would allow for a lower b^*_1 so that the only equilibrium is G. The same happens if we start to the left of G, with i and b^* increasing up to G. On the other hand, if B exists, it should be that $\partial \text{IR}/\partial b^*|_B > \partial \text{GR}/\partial b^*|_B$, which means that B is not an attractor. The reader can easily see that for any arbitrary i_0 to the right of B the subsequent values along IR and GR would deviate from B because the market would want a higher i which would require a higher b^* and so on up to i^D, which requires $\bar{b}$ and hence triggers default. Hence, to the right of B the government is bound to default. We can therefore establish the following proposition:

(P1) a) *If a good and bad equilibrium exist, the good equilibrium is an attractor for any initial condition below the bad equilibrium.* b) *For any initial condition above the bad equilibrium, the government is bound to default.*

In the light of (P1), the key issue is to establish the extent of the good-equilibrium domain, or in President Draghi's words, how good the fiscal outlook should be in order to remain within the good-equilibrium domain. Our previous analysis shows that there is no clear-cut answer: the extension

of the good (or bad) equilibrium domain depends on the characteristics of *both* the GR *and* IR functions. The shape and location of the IR function are crucial factors, and they depend on the mean and variance of the distribution of beliefs, a typical important feature of HB models (Kurz (2011)). In other words, *fiscal fundamentals and market beliefs cannot be disentangled.* It is convenient to examine these issues by putting the model at work, with also a view to some problems that are currently under discussion in connection with the EZ sovereign crisis.

3. The model at work

3.1. Fiscal and market shocks, fundamentals and non-fundamentals

To being with, let us examine how, given an initial good equilibrium, the system reacts to shocks. The model can deal with both *fiscal* and *market* shocks. The former affect the GR function, the latter affect the IR function. Concomitantly, the model, albeit stylised, may help shed some light on the much-debated issue of the role of fundamentals *vs.* non-fundamentals. The fundamentals are captured by the GR function. Market assessment is captured by the IR function which may react to fundamental as well as non-fundamental news. Fiscal and market shocks can, of course, compound.

The outcome of any type of shock eventually depends on the initial position and on the new configuration of the two functions. The key issue is whether or not a new set of fixed points exists. If it exists, then the system possess a new good equilibrium; otherwise the government is bound to default.

Fiscal shocks

We can consider two types of (negative) fiscal shocks that shift the GR function outwards. The first type is due to a fall in nominal GDP, $z < 0$, the second is due to extraordinary operations that rise debt *coeteris paribus*, $x > 0$ (see equation (4) that traces the transitional phase of the solvency condition). Recall that these news are common knowledge at the time of budget planning t. Also note that, although the GR function represents solvency sustainability in steady states, once the debt market is shifted away from an equilibrium, the maturity structure of debt does matter along the transitional phase; in fact, as the interest rate increases, its impact on

solvency is filtered through the rate of expirations and rollovers. If short-maturity debt is high, e is high and entails a stronger impact of the current interest rate on θ_t and on solvency (i.e. a faster transition towards new equilibrium or default).

Let us examine the two cases portrayed in Figure 5. Case a) exemplifies a "small" negative shock. Starting at the good equilibrium G, the government commits itself to a higher b^* against which the market sets a higher i^*. The new good equilibrium is G_1. If instead the government starts at the bad equilibrium B, the small shock is sufficient to lead to default.

Figure 5a. A small fiscal shock
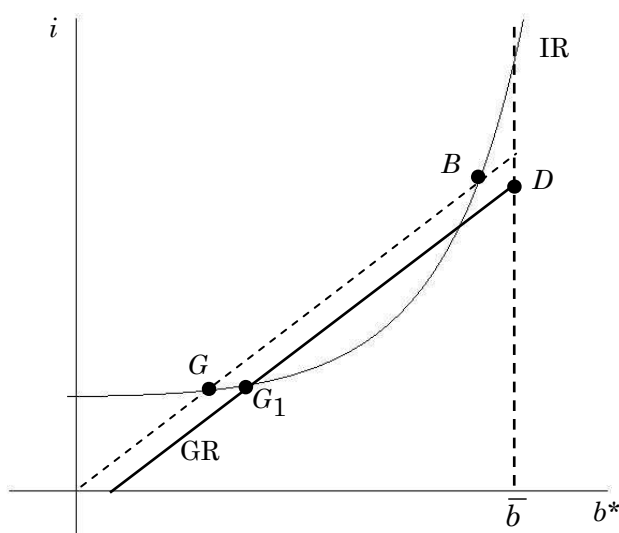
It is worth considering the system in the neighbourhood of G in some detail. Note that in G_1 both b^* and i^* are eventually higher than they would have been in the absence of an increase in the market probability of default (the movement along the IR function). However, we know that in the region of low b^* and flat IR the increase in i may be small or negligible. It becomes substantial only as b^* approaches and then exceeds $\bar{b}_M$.

This is a noteworthy feature that can shed some light on one of the several puzzles that have recently emerged in the studeis on risk premia. Why did spreads across EZ sovereign debts remain so small until 2009 regardless of differences in debt stocks and deficits? Why do the United States or the United Kingdom or Japan pay negligible spreads in comparison with not so fiscally worse (or even better) EZ sovereigns? As said above, this model suggests that fiscal fundamentals (debt stocks, deficits,

shocks, etc.) do not matter *per se* but in relation to the configuration of the IR function, and in particular the distance of b^* from the market belief $\bar{b}_M$ about the true $\bar{b}$. A high $\bar{b}_M$ allows the government to sustain a larger b^* with lower default premium.

Rational beliefs require that $\bar{b}_M$ is consistent with the true $\bar{b}$, and the government's choice model says that the latter is higher when the cost of default is higher or the cost of solvency is lower. The confidence of holders of EZ sovereign debts before the crisis in a *de facto* lender of last resort (as against the official no-bailout clause of the Treaties) and in the no exit option are factors that, respectively, reduce the cost of solvency and rise the cost of default, and thus justify a high $\bar{b}_M$. Confidence in these two factors has been shaken during the crisis management. On the other hand, according to De Grauwe and Ji (2012), stand alone sovereigns pay comparatively lower risk premia because they can eventually rely on their own lender of last resort. In our terms, this in fact entails a lower cost of solvency, and hence higher $\bar{b}_M$ and flatter IR, than EZ sovereigns.

Case b) exemplifies a "large" negative shock, e.g. a bailout of banks $x > 0$, as in Ireland or Spain, that shifts the GR function below the IR function so that no new fixed points exist.

Figure 5b. A large fiscal shock
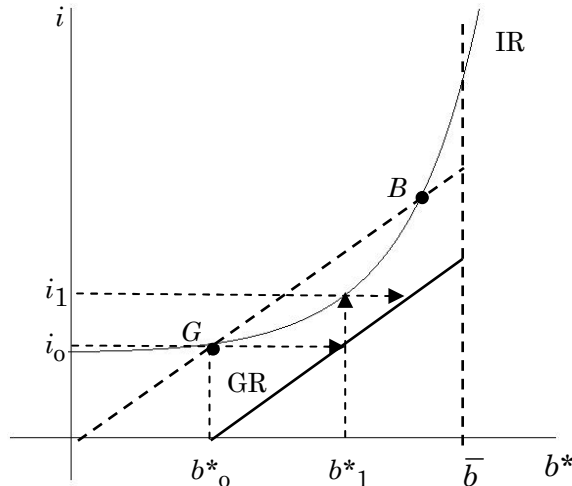
Starting at i_0, the government can commit itself to, and can sustain, b^*_1, but the market would raise the interest rate to i_1, which would require a higher b^*, and so on up to default. This is a typical case of a SF, non-fundamental attack, because b^*_1 is sustainable by the government, and from that point

onwards nothing changes in fundamentals that justifies the attack except self-generated higher interest payments.

We have seen the events of SF beliefs of default. A connection exists with the well-known phenomenon of liquidity problems leading to insolvency crises. Technically speaking, as long as $b^* < \bar{b}$ the government is able and willing to refrain from additional borrowing. Yet, expirations need liquidity by means of rollovers, and as is often the case, the indicator of the problem is the increasing interest rate on rollovers, which in the long run has to be paid on whole debt stock (see above 2.1). As explained above, the movement along the IR curve is driven by a larger and larger fraction of investors who wish to sell their stock of bonds facing a smaller and smaller fraction willing to buy. The cost of rollovers grows because for sellers the market liquidity shrinks. As previously warned, at this stage the model does not accommodate a formal analysis of the system's behaviour out of equilibrium. However, the presence of heterogeneous investors is crucial, otherwise little can be said about market adjustments. The transition towards default may be slower or quicker depending on the location and shape of the IR function, that is, the underlying distribution of beliefs. A sort of comparative-static analysis of changes at this level is provided in the next paragraph.

Market shocks

The model can also deal with market shocks, that is, changes in the belief distribution and hence in the IR function. Changes may be provoked by an external factor or arise endogenously in the market process (learning, imitation, etc.). For the reasons already said, endogenous changes will not be addressed here, but it is possible to see the effects indirectly. The point is how these changes affect market conditions faced by the government, where the former are identified by the mean and variance of the belief distribution.

For a given state of GR, the attraction domain of the good equilibrium may shrink owing to (i) lower market belief $\bar{b}_M$ or (ii) lower variance of beliefs ("belief coagulation"). As an example, Figure 6 shows the IR functions generated by Normal distributions that differ in their mean and variance. Take N~(7, 1.4) as benchmark. A lower $\bar{b}_M$ makes the IR steeper; a lower variance has an interesting two-faceted effect: the IR is flatter below $\bar{b}_M$ (in fact the mass of pessimists (low $\tilde{b}$) is reduced), and it is steeper above $\bar{b}_M$ (because the mass of optimists (high $\tilde{b}$) is reduced too). Anyway,

the bad-equilibrium point shifts downwards, and the attraction domain of the good equilibrium shrinks.

Figure 6. The IR function with different Normal distributions of investors' beliefs.
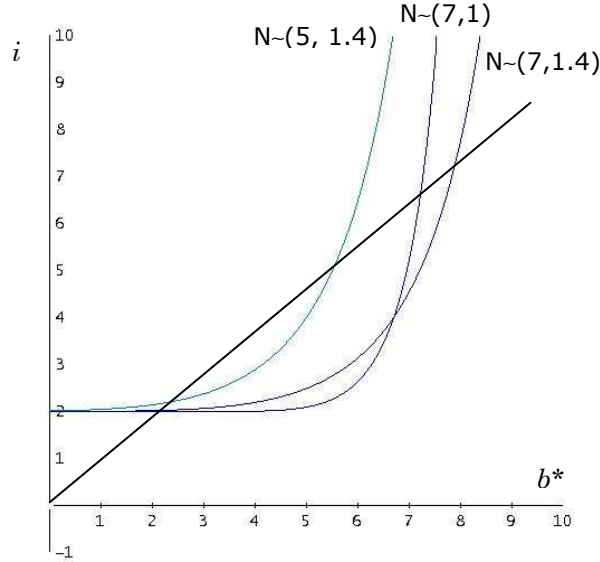
This role of the mean and variance of beliefs also explains, and supports, two important features of the model regarding information available to individual investors. First, governments usually do not communicate their true $\bar{b}$ because they have no incentive to do it. On the one hand, the mean effect indicates that governments have an incentive to communicate a value of $\bar{b}$ greater than the true one, or that they will *never* default, which makes their communication worthless to investors. On the other hand, the variance effect shows that dispersed beliefs create more favourable conditions precisely when b^* is relatively high. Second, observed couples (b^*, i) cannot consistently reveal the underlying distribution because two different distributions may generate the same (b^*, i).

As long as investors form their beliefs rationally, however, news about changes in the determinants of $\bar{b}$ will be reflected by $\bar{b}_M$ consistently. Think again of the role of beliefs about EZ governments' $\bar{b}$ before and after the crisis. The discovery that the no-bailout clause may be effective, and that the exit option has nonzero probability, entails that governments' cost-comparative analysis tilts towards lower $\bar{b}$. As shown by Figure 6, the IR with lower $\bar{b}_M$ shifts inwards determining more severe market conditions *coeteris paribus.* Yet these shifts may be related to fundamentals or not. If news that Greece is closer to default feed the same belief as to Italy, so that

its $\bar{b}_M$ is lowered, this can be classified as contagion of non-fundamentals. For beliefs may prevail over reality as the government, *coeteris paribus*, finds itself in the attraction domain of default in spite of the fact that its true $\bar{b}$ is higher than believed by the market.

Changes in the variance of beliefs relate to another well-known, controversial question: the role of the degree of heterogeneity of beliefs. If for some reason beliefs coagulate around the mean, we have seen that the effect is twofold. For low $b*$ the market conditions are less severe (IR is flatter); but for high $b*$ the market conditions are more severe (IR is steeper). Therefore, we cannot say a priori whether coagulation of beliefs is beneficial or detrimental: it depends on the fiscal outlook of the country. However, we can say that the coagulation of beliefs is a self-sustaining mechanism, low $b*$ states are further eased thanks to a smaller mass of pessimists, while high $b*$ states are further tightened owing to a smaller mass of optimists. These effects also cast a problematic light on external agents that may foster coagulation of beliefs, such as the role of opinion makers, official institutions, rating agencies, *gurus*, etc. And these factors, too, may be related or unrelated to fundamentals up to panic episodes such that beliefs suddenly coagulate around a low $\bar{b}_M$ creating extremely severe solvency conditions.

3.2. Foreign debt *vs.* domestic debt, and the "Grexit" option

How the composition of debt affects investors' appetite is matter of extensive research, but the issue has recently been raised in the context of the crisis of the EZ sovereign debt, pointing out a relationship among persistent current account deficits, accumulation of foreign debt and higher risk premia (Gros (2011, 2013), Alessandrini et al. (2012)). This relationship is, however, controversial (Obstfeld (2012)). My aim here is not to take a position but to show how the discussion can be clarified within the present framework.

One controversial issue is why a larger share of foreign debt should come with a higher interest rate. According to the present model, a higher interest rate may be the result of either worse fundamentals or worse market beliefs. As to fundamentals, e.g. growth capacity, there is no clear connection with the composition of debt. Hence the problem lies in the way the foreign component of debt affects the IR function.

The problem can be addressed from two different viewpoints: that of investors in general, and that of foreign investors in particular. As to investors in general, one argument is that the presence of foreign debt restricts the government's ability to service its debt because foreign investors cannot be taxed. Since the tax burden would fall on the sole shoulders of domestic taxpayers, the government's solvency costs, both economic and political, would be higher. As a consequence, the market belief $\bar{b}_M$ would be lower, which, as seen above, would determine a higher interest rate *coeteris paribus*.

As to foreign investors, an oft-heard argument is that they may fear the so-called "selective default". If the government could default on foreign debt only, the default costs would be reduced. This conjecture, too, lowers $\bar{b}_M$ and concentrates the risk on foreigners at the one and same time. A complementary argument is that foreign capital is typically more volatile than domestic capital, so that fire sales of the government bonds would be fast and large. However, selective default in a highly integrated financial system of cross-border private investors mixed up with large multinational entities is technically and legally quite problematic (e.g. private foreign investors may hold shares in resident investment funds holding domestic debt).

A particular specific risk faced by foreign investors is currency devaluation, which operates as a (partial) haircut of the value of government's payments due to foreign investors. Hence, in general, currency risk rises the risk premium, but in the case of EZ sovereigns this problem has the peculiar aspect that there is no longer currency sovereignty. In normal times, debt denominated in euros offers total protection to *all EZ resident investors*, so what is relevant is only the share of *non-EZ non-resident investors* and the chance of devaluation of the euro, which is not under direct control of single governments. If this factor plays any role, it should materialize uniformly in all EZ countries interest rates *vis-à-vis* non-EZ countries. In fact, data suggest that until 1998 interest-rate differentials across the subsequent EZ countries were largely driven by the currency risk component, which almost disappeared with the monetary union (Wyplosz (2006)).

However, the relevance of foreign debt in the EZ debt crises has been used to explain differentials *within* the EZ. If currency risk may play a role, it should be *country-specific* currency risk, which can only be related to the

event of exit from the monetary union followed by devaluation of the newborn national currency – the so-called "Grexit" option. This event can in turn be conceived of as an *alternative* to technical default. Hence Grexit is actually a selective partial haircut on foreign obligations by way of devaluation, which saves domestic investors. As argued above, selective default lowers $\bar{b}_M$ and raises the IR function, but the Grexit option is highly costly and pushes in the opposite direction. This possibility complicates the picture substantially because the population of investors is split between *all* foreign *vs.* domestic ones. Let $F(b^*)$ be the fraction of *all* investors who believe that the government is going to default like before, but now the government may also choose between No-exit and Grexit. In the former case, all investors will be treated equally and we are back in the previous treatment. In the latter case, only foreign investors will pay a fee in association with the *joint* event of $b^* > \bar{b}$ *and* Grexit (which may however be less severe than technical default in the No-exit case). For the foreign debt to exert significant effect on spreads via currency risk, the probability of the Grexit option should be non-negligible *and* the share of *pessimist* foreign investors should be large. Therefore, a critical factor, as it turned out to be during the crisis, is the *belief in the irreversibility of the euro.* As a matter of fact, the ECB's complaints for excessive spread for specific countries was mostly attributed to rumours about their exit from the EMU (Draghi (2012)).

Overall, the present model suggests that a high share of foreign debt may create adverse market conditions, but the actual effect depends not so much on foreign debt *per se* as on its interaction with other economic and institutional factors that shape *the investors' beliefs*.[16]

3.3. Why "austerity" may not work: A model of the "Greek tragedy"

The traditional "shock therapy" of front-loaded, "ambitious" fiscal consolidation plans, also known in Europe as "austerity" has now become

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[16] According to a recent research by Arslanalp and Poghosyan (2014), the quantitative impact of the foreign share on the sovereign bond yields in a sample of advanced economies in 2008-12 does not seem very large. They have found that one euro more (less) of foreign investment entails 6-10 basis points less (more) in the yield. For a country with about 50% of foreign debt like Italy, foreign outflows have accounted for 40-70 basis point in an escalation of about 450 points.

highly controversial well beyond the circles of traditional opponents.[17] Greece, Portugal and Ireland have undergone "Troika" shock therapies as conditionality for access to rescue funds. The governments of Italy and Spain in power since 2011 have sought to follow the same strategy pre-emptively. Notwithstanding hard austerity plans, these countries have experienced persistently high (or increasing) high spreads. The fever of high spreads started to recede in the last quarter of 2012 only after the ECB launched the new OMT programme. These facts raise the thorny issue whether such plans were too small (non credible) or too large (non sustainable). On the other hand, the absence, or belated and poor, design of rescue packages has also been criticized. The GR-IR model can provide an analytical treatment of these problems.

From the initial good equilibrium G, let us consider again a large fiscal shock such that the GR function shifts to GR1 into the default domain as in Figure 7.

Figure 7. A model of the "Greek tragedy"
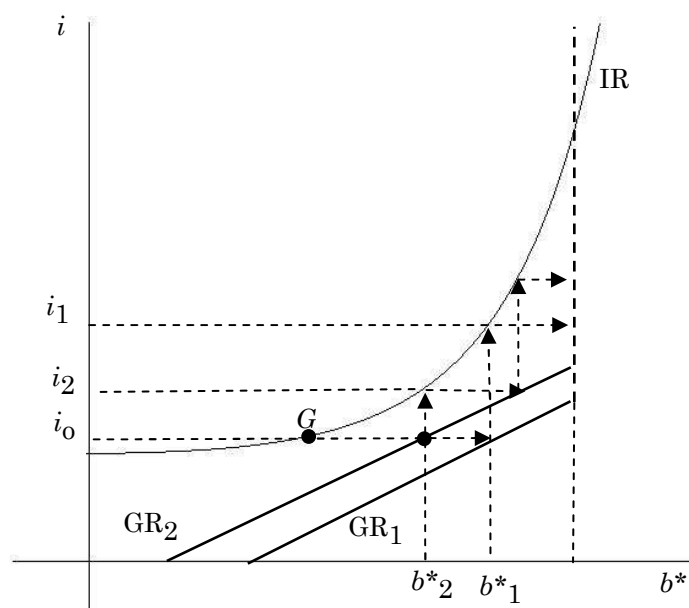
At the initial interest rate i_0, the government is still ready to stay solvent with b^*_1. However, at b^*_1 the interest rate would rise to i_1, and the government would eventually default. Hence the government instead files for a rescue package, say some loan with conditionality, which shifts the

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[17] See among others the *Forum* organized by the website *Vox* (www.voxeu.org) and Corsetti (ed., 2012). For an overall assessment see Tamborini (2014).

GR_1 function leftwards to GR_2.[18] The conditionality commits the government to achieving any solvency primary surplus dictated by "market discipline".

The model clarifies that whether the loan + austerity package is good or bad cannot be judged independently of the context. One critical factor is whether the loan is sufficient to reach the good-equilibrium domain. Suppose it is not, as shown in the figure. At the initial interest rate i_0, the new solvency primary surplus is reduced to b^*_2, but then we observe the following notable events. As the government commits itself to b^*_2, the market would respond with an increase in the interest rate to i_2, which again sets the government on an unsustainable path. *Increasing fiscal effort is the wrong policy to tame the market in the default attraction domain.* Hence, it is the combination of austerity with an insufficient loan that condemns the rescue package to failure. Note that we have obtained this outcome with no depression effects of austerity on GDP, which would exacerbate the problem (the GR function would shift rightwards). This sequence of events is remarkably resembling what happened with the so-called "Greek tragedy".

The lesson to be drawn is twofold: first, the market response to the plan is not part of the solution but part of the problem; second, a successful rescue plan should be *large* and *concessional* enough to pull the government out of the default domain. Ideally, there are two possible solutions. The first could be dubbed the "market-based" solution, that is, the smallest loan consistent with regaining a market interest rate sustainable by the government. This is tantamount to shifting the GR_2 function up to the tangency point with IR in a new (single) equilibrium.[19] This "tangency solution" is the same policy recommendation offered by Corsetti and Dedola (2011) and Cooper (2012). However, it is by no means easy to engineer, not least because the IR function is not easily detectable as seen in the course of the Greek tragedy.

The second solution is more straightforward and consists of setting a ceiling on the interest rate. The lending institution may charge a

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[18] Different formats of the rescue package can be accommodated in the model. Direct conditional loans, such as those granted by the IMF or the newly created European Stability Mechanism (ESM), are captured by $x < 0$, which cuts b^*. The same effect obtains with an ECB intervention in the secondary debt market.

[19] Technically, this would not be a stable equilibrium. However, a small additional leftward shift of GR would provide a new good equilibrium.

concessional interest rate (e.g. i_0) with sustainable conditionality (b^*_2), which prevents the start-up of the SF attack. This can be accomplished by absorbing the excess supply of bonds of the pessimist investors for whom $b^*_2 > \tilde{b}$. From the search-of-equilibrium point of view, there is a clear advantage in the direct negotiation of two single institutions. The same point was made by De Grauwe (2011), who in this perspective criticized the operation rules of the EFSF as well as of the then forthcoming ESM for both being too reliant on market rates and not taking into account sustainability. From this viewpoint, the OMT programme, to the extent that it implies an interest-rate ceiling on the applicant's sovereign debt, appears as a superior solution, *provided that the conditional part is sustainable*. The complaint that the central bank takes undue risks appears to be groundless in this picture, both because it has the powers to remove uncertainty on the sustainability of the fiscal plan and because *the concessional interest rate itself generates the conditions for sustainability*.

4. Conclusions

The dramatic EZ sovereign debt crisis has prompted a new generation of models of debt dynamics and management characterized by multiple equilibria (ME) due to interactions between fiscal fundamental variables and investors' assessment of default probability. Typically, these interactions may give rise to SF attacks on the sovereign debtor, leading to default in spite of initial sustainable conditions. In this paper I have presented a ME model in this vein, whose main novel feature is the presence of HB of investors regarding the threshold level of fiscal solvency effort (the primary surplus/GDP ratio) at which the government opts for default. Relatedly, the default probability is not attributed to a single representative investor but it is measured as the cumulated fraction of investors who believe that the fiscal solvency effort to which the government has committed itself exceeds its default threshold. The model identifies an attraction domain of default within which the government is bound to default although initial solvency conditions are sustainable. The extent of this domain may be larger or smaller depending on the interplay between fiscal fundamentals and the *distribution* of investors' beliefs.

By means of this model some controversial issues have been addressed in the current debate on the EZ sovereign debt crises, such as puzzles

concerning the pattern of risk premia before and after the crisis, the identification of non-fundamental and contagion components in risk premia, the role of the foreign component of debt, pitfalls in "austerity" therapies.

Some relevant policy implications also ensue. First, it is crucial that fundamental as well as non-fundamental cross-country interdependencies are taken into account in the policy design. Second, rescue systems should be in place against the default attraction domain. In fact, it is hard for a government to escape from this domain by its own means. In particular, in this domain austerity may not be the right response, even ignoring possible contractionary effects on GDP. For "ambitious" fiscal plans are assessed as unsustainable by a larger share of investors bidding for higher, not lower, interest rate. Among rescue systems, both central banks' interventions in the sovereign-debt market and bailout packages may be effective provided that they are large enough to remove the country's fiscal outlook from the default domain. Both instruments, implicitly or explicitly, entail the charging of a concessional interest rate as long as necessary. Against this background, the original EMU institutional setup has been part of the problem rather than of the solution. The newly created rescue mechanisms, except perhaps the OMT, though untested, do not seem up to the task.

The purpose of this paper was to show that the heterogeneity hypothesis allows for a richer and more realistic analysis of ME systems with respect the representative agent and RE equilibria. A more general message is that phenomena in contrast with the efficient market paradigm need not be confined in the non scientific realm of irrationality, but they can be understood, and to some extent prevented and controlled, by changing the paradigm. Yet this was a first step of analysis based on strict (simplifying) assumptions regarding the formation of beliefs and limited to the existence and characterization of ME in relation to given distributions of beliefs. Integration of a formal analysis of the system's behaviour out of equilibrium as well as the extension of formation of beliefs to include for instance second order beliefs on the beliefs of others, or endogenous modifications of beliefs in the market process, are important refinements that are left for further developments.

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